FIITJEE

KUKATPALLY CENTRE

IMPORTANT QUESTIONS

FOR

INTERMEDIATE PUBLIC EXAMINATIONS

IN

MATHS-IIB

2017-18

INTERMEDIATE PUBLIC EXAMINATION, MARCH 2017

Total No. of Questions - 24 **Reg. No.**

Total No. of Printed Pages - 2

Part - III

MATHEMATICS, Paper-II (B)

(English Version)

Time : 3 Hours] **[Max. Marks : 75**

SECTION - A $10 \times 2 = 20$ M

I. Very Short Answer Type questions:

1. Obtain the parametric equation of the circle $4(x^2+y^2)=9$.
2. Find the value of k, if the points (4, 2) and $(k,-3)$ are conjugate points with respect to the circle $x^2+y^2-5x+8y+6=0$.
3. Find the angle between the circles given by the equations $x^2+y^2-12x-6y+41=0$, $x^2+y^2+4x+6y-59=0$.
4. Find the coordinates of the points on the parabola $y^2=8x$ whose focal distance is 10.
5. If $3x-4y+k=0$ is a tangent to the hyperbola $x^2-4y^2=5$. Find the value of k.
6. Evaluate $\int \frac{1}{\cosh x+\sinh x}dx$ on R.
7. Evaluate $\int \frac{e^x(1+x)}{\cos^2(xe^x)}dx$ on $I \subset R \setminus \{x \in R : \cos(xe^x)=0\}$
8. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin|x|dx$
9. Evaluate $\int_0^3 \frac{x}{\sqrt{x^2+16}}dx$
10. Find the order of the differential equation of the family of all the circles with their centres at the origin.

SECTION – B $5 \times 4 = 20$ M

II. Short Answer Type questions:

(i) Attempt any **five** questions

(ii) Each question carries **four** marks

11. If a point P is moving such that the lengths of tangents drawn from P to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$ are in the ratio 2 : 3 then find the equation of the locus of P.
12. Find the equation and the length of the common chord of the following circles: $x^2+y^2+2x+2y+1=0$; $x^2+y^2+4x+3y+2=0$
13. Find the equation of ellipse in the standard form, if it passes through the points $(-2,2)$ and $(3,-1)$.

14. Find the equation of the tangents to the ellipse $2x^2 + y^2 = 8$ which are

(i) parallel to $x - 2y - 4 = 0$ (ii) perpendicular to $x + y + 2 = 0$

15. If e, e_1 are the eccentricities of a hyperbola and its conjugate hyperbola prove that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$.

16. Find the area of the region bounded by the parabola $y^2 = 4x$ and $x^2 = 4y$.

17. Solve the following differential equation $(x + y + 1)\frac{dy}{dx} = 1$

SECTION – C $5 \times 7 = 35$ M

III. Long Answer Type questions:

(i) Attempt any **five** questions

(ii) Each question carries **seven** marks

18. If $(2,0)$, $(0,1)$, $(4,5)$ and $(0,c)$ are concyclic the find c.

19. Find the transverse common tangents of the circles $x^2 + y^2 - 4x - 10y + 28 = 0$ and $x^2 + y^2 + 4x - 6y + 4 = 0$.

20. Derive the equation of a parabola in the standard form $y^2 = 4ax$ with diagram.

21. Evaluate $\int \frac{9\cos x - \sin x}{4isnx + 5\cos x}dx$.

22. If $I_n = \int \cos^n x dx$, then show that $I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$ and for $n \geq 2$ deduce the value of $\int \cos^4 x\, dx$.

23. Show that $\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx = \frac{\pi}{2\sqrt{2}} \log(\sqrt{2} + 1)$.

24. Solve the differential equation $(x - y)dy = (x + y + 1)dx$

BLUE PRINT (MATHS-IIB)

S.No.	Chapter Name	Weightage Marks
Coordinate Geometry		
1.	Circles	15 (2 + 2 + 4 + 7)
2.	System of Circles	13 (2 + 4 + 7)
3.	Parabola	9 (2 + 7)
4.	Ellipse	8 (4 + 4)
5.	Hyperbola	6 (2 + 4)
Calculus		
6.	Indefinite Integration	18 (2 + 2 + 7 + 7)
7.	Definite Integration & Areas	15 (2 + 2 + 4 + 7)
8.	Differential Equations	13 (2 + 4 + 7)

VERY SHORT ANSWER QUESTIONS

1.

◆A. Find the equation of circle with centre (1, 4) and radius 5.

◆B. Find the centre and radius of the circle $x^2+y^2+2x-4y-4=0$

◆C. Find the centre and radius of the circle $3x^2+3y^2-6x+4y-4=0$

◆D. Find the equation of the circle whose centre is (-1, 2) and which passes through (5, 6)

◆E. Find the equation of the circle passing through (2, 3) and concentric with the circle $x^2+y^2+8x+12y+15=0$

◆F. If the circle $x^2+y^2+ax+by-12=0$ has the centre at (2, 3) then find a, b and the radius of the circle.

◆G. Find the equation of the circle whose extremities of a diameter are (1, 2) and (4, 5)

◆H. Find the other end of the diameter of the circle $x^2+y^2-8x-8y+27=0$ if one end of it is (2, 3)

◆I. Obtain the parametric equation of the circle represented by $x^2+y^2+6x+8y-96=0$

J. Find the values of a,b if $ax^2+bxy+3y^2-5x+2y-3=0$ represents a circle. Also find the radius and centre of the circle.

K. Find the centre and radius of each of the circles $\sqrt{1+m^2}\left(x^2+y^2\right)-2cx-2mcy=0$

L. Obtain the parametric equation of each of the following circles.

(i) $x^2+y^2=4$ (ii) $(x-3)^2+(y-4)^2=8^2$

M. Show that $A(3,-1)$ lies on the circle $x^2+y^2-2x+4y=0$. Also find the other end of the diameter through A.

2.

◆A. Locate the position of the point (2, 4) with respect to the circle $x^2+y^2-4x-6y+11=0$.

◆B. Find the length of the tangent from (1, 3) to the circle $x^2+y^2-2x+4y-11=0$.

◆C. Find the equation of the tangent to $x^2+y^2-6x+4y-12=0$ at $(-1,1)$

◆D. If the parametric values of two points A and B lying on the circle $x^2+y^2-6x+4y-12=0$ are $30°$ and $60°$ respectively then find the equation of the chord joining A and B.

◆E. Find the area of the triangle formed by the normal at $(3,-4)$ to the circle $x^2+y^2-22x-4y+25=0$ with the coordinate axes.

◆F. If $(4,k)$ and $(2,3)$ are conjugate points with respect to the circle $x^2+y^2=17$ then find k.

◆G. Find the power of the point P with respect to the circle $S=0$ when

(i) $P=(5,-6)$ and $S\equiv x^2+y^2+8x+12y+15$

◆H. If the length of the tangent from (2, 5) to the circle $x^2+y^2-5x+4y+k=0$ is $\sqrt{37}$ then find k.

◆I. Find the length of the chord formed by $x^2+y^2=a^2$ on the line $x\cos\alpha+y\sin\alpha=p$.

◆J. Find the value of k if the points $(4,2)$ and $(k,-3)$ are conjugate points with respect to the circle $x^2+y^2-5x+8y+6=0$.

K. Find the equation of the normal to the circle $x^2+y^2-4x-6y+11=0$ and $(3,2)$. Also find the other point where the normal meets the circles.

L. Find the equation of the pair of tangents from $(10,4)$ to the circle $x^2+y^2=25$.

M. Find the equation of the normal at P of the circles $S=0$ where P and S are given by

(i) $P=(3,-4)$, $S=x^2+y^2+x+y-24$

N. Fin the polar of $(1,2)$ with respect to $x^2+y^2=7$.

O. Find the pole of $ax+by+c=0$ $(c\neq 0)$ with respect to $x^2+y^2=r^2$.

P. Find the pole of $3x+4y-45=0$ with respect to $x^2+y^2-6x-8y+5=0$

Q. Find the value of k if the point $(1,3)$ and $(2,k)$ are conjugate with respect to the circle $x^2+y^2=35$.

R. Find the angle between the tangents drawn from $(3,2)$ to the circle $x^2+y^2-6x+4y-2=0$.

S. Find the number of possible common tangents that exist for the following pairs of circles

$x^2+y^2+6x+6y+14=0$, $x^2+y^2-2x-4y-4=0$

3.

◆A. If the angle between the circles $x^2+y^2-12x-6y+41=0$ and $x^2+y^2+kx+6y-59=0$ is $45°$ find k.

◆B. Find k if the following pairs of circles are orthogonal

(i) $x^2+y^2+2by-k=0$, $x^2+y^2+2ax+8=0$

◆C. Find the angle between the circles given by the equations

$x^2+y^2-12x-6y+41=0$, $x^2+y^2+4x+6y-59=0$

◆D. Show that angle between the circles $x^2+y^2=a^2$, $x^2+y^2=ax+ay$ is $\frac{3\pi}{4}$.

◆E. Find the equation of the common tangent of the following circles at their point of contact

$x^2+y^2+10x-2y+22=0$, $x^2+y^2+2x-8y+8=0$

F. Find the equation of the radical axis of the following circles

◆(i) $x^2+y^2-2x-4y-1=0$, $x^2+y^2-4x-6y+5=0$

(ii) $x^2+y^2+2x+4y+1=0$, $x^2+y^2+4x+y=0$

G. Find the equation of the common chord of the following pair of circles.

(i) $x^2+y^2-4x-4y+3=0$, $x^2+y^2-5x-6y+4=0$

◆(ii) $(x-a)^2+(y-b)^2=c^2$, $(x-b)^2+(y-a)^2=c^2\,(a\neq b)$

4.

◆A. Find the equation of the parabola whose vertex is $(3,-2)$ and focus is $(3,1)$.

◆B. Find the coordinates of the points on the parabola $y^2=2x$ whose focal distance is $\frac{5}{2}$.

◆C. Find the vertex and focus of $4y^2+12x-20y+67=0$.

◆D. Find the equation of the parabola whose focus is $S(1,-7)$ and vertex is $A(1,-2)$.

◆E. Find the coordinates of the points on the parabola $y^2 = 8x$ whose focal distance is 10.

◆F. If (1/2, 2) is one extremity of a focal chord of the parabola $y^2 = 8x$. Find the coordinate of the other extremity.

◆G. Find the value of k if the line $2y = 5x + k$ is a tangent to the parabola $y^2 = 6x$.

◆H. Find the equation of the normal to the parabola $y^2 = 4x$ which is parallel to $y - 2x + 5 = 0$.

I. Find the vertex and focus of $x^2 - 6x - 6y + 6 = 0$.

J. Find the equations of axis and directrix of the parabola $y^2 + 6y - 2x + 5 = 0$.

K. Find the equation of the parabola whose focus is $S(3,5)$ and vertex is $A(1,3)$.

L. Find the equation of the parabola whose latus rectum is the line segment joining the points $(-3,2)$ and $(-3,1)$.

M. Find the equations of the tangent and normal to the parabola $y^2 = 6x$ at the positive end of the latus rectum.

N. Find the equation of the tangent and normal to the parabola $x^2 - 4x - 8y + 12 = 0$ at $\left(4, \frac{3}{2}\right)$.

O. Show that the line $2x - y + 2 = 0$ is a tangent to the parabola $y^2 = 16x$. Find the point of contact also.

P. Find the equation of tangent to the parabola $y^2 = 16x$ inclined at an angle 60° with its axis and also find the point of contact.

5.

◆A. If e, e_1 are the eccentricities of a hyperbola and its conjugate hyperbola prove that $\frac{1}{e^2} + \frac{1}{e_1^2} = 1$

◆B. Find the equations of the hyperbola whose foci are $(\pm 5, 0)$, the transverse axis is of length 8.

◆C. If $3x - 4y + k = 0$ is a tangent to $x^2 - 4y^2 = 5$ find the value of k.

◆D. If the eccentricity of a hyperbola is $\frac{5}{4}$, then find the eccentricity of its conjugate hyperbola.

◆E. Find the equation to the hyperbola whose foci are (4, 2) and (8, 2) and eccentricity is 2.

F. If the $lx + my = 1$ is a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then, show that $\frac{a^2}{l^2} - \frac{b^2}{m^2} = (a^2 + b^2)^2$

G. Find the equations of the tangents to the hyperbola $3x^2 - 4y^2 = 12$ which are

(i) parallel and (ii) perpendicular to the line $y = x - 7$

H. Define rectangular hyperbola. What is its eccentricity?

I. Find the product of lengths of lengths of the perpendiculars from any point on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ to its asymptotes.

J. Find the equation of the hyperbola whose asymptotes are $3x = \pm 5y$ and the vertices are $(\pm 5, 0)$.

K. Find the equation of the normal at $\theta = \pi / 3$ to the hyperbola $3x^2 - 4y^2 = 12$.

L. If the angle between the asymptotes is 30^0 then find its eccentricity.

6.

◆A. Evaluate $\int \left(x+\frac{1}{x}\right)^3 dx\ ,\ x>0$

◆B. Find $\int \sqrt{1+\sin 2x}\ dx$ on $\mathbb{R}$.

◆C. Evaluate $\int \frac{x^5}{1+x^{12}}\ dx$ on $\mathbb{R}$

◆D. Find $\int \left(1-\frac{1}{x^2}\right).e^{\left(x+\frac{1}{x}\right)}\ dx$ on where $I=(0,\infty)$

◆E. Evaluate $\int \frac{dx}{(x+5)\sqrt{x+4}}\ dx$ on $(-4,\infty)$

◆F. $\int \frac{(a^x-b^x)^2}{a^x b^x}\ dx$, $(a>0,\ a\neq 1$ and $b>0,\ b\neq 1)$ on $\mathbb{R}$

◆G. $\int \sec^2 x \operatorname{cosec}^2 x\ dx$ on $I\subset \mathbb{R}\setminus\left(\{n\pi : n\in\mathbb{Z}\}\cup\left\{(2n+1)\frac{\pi}{2} : n\in\mathbb{Z}\right\}\right)$

◆H. $\int \sqrt{1-\cos 2x}\ dx$ on $I\subset[2n\pi,\ (2n+1)\pi],\ n\in\mathbb{Z}$

◆I. $\int \frac{1}{\cosh x+\sinh x}\ dx$ on $\mathbb{R}$.

J. Evaluate $\int \cot^2 x\,dx$ on $\subset \mathbb{R}\setminus\{n\pi : n\in\mathbb{Z}\}$

K. Evaluate $\int \left(\frac{x^6-1}{1+x^2}\right)dx$ for $x\in\mathbb{R}$.

L. Evaluate $\int \cos^3 x \sin x\ dx$ on $\mathbb{R}$

M. Evaluate $\int \frac{\sin^4 x}{\cos^6 x}\ dx\ ,\ x\in 1\subset R\setminus\left\{\frac{(2n+1)\pi}{2} : n\in\mathbb{Z}\right\}$

N. Evaluate $\int \sin^2 x\ dx$ on $\mathbb{R}$

O. Evaluate $\int \frac{dx}{\sqrt{4-9x^2}}$ on $I=\left(-\frac{2}{3},\frac{2}{3}\right)$

P. Evaluate $\int \frac{1}{1+4x^2}\ dx$ on $\mathbb{R}$

Q. Evaluate $\int e^{\log(1+\tan^2 x)}dx$ on $I\subset\mathbb{R}\setminus\left\{\frac{(2n+1)\pi}{2} : n\in\mathbb{Z}\right\}$

R. Evaluate $\int \frac{\sin^2 x}{1+\cos 2x}dx$ on $I\subset\mathbb{R}\setminus\{(2n\pm 1)\pi : n\in\mathbb{Z}\}$

S. Evaluate $\int \frac{1}{1+\cos x}\ dx$ on $I\subset\mathbb{R}\setminus\{(2n+1)\pi : n\in\mathbb{Z}\}$

T. Evaluate $\int \frac{x^2}{1+x^2}\ dx$ on $\{I\subset\mathbb{R}\setminus 0\}$

7.

◆A. $\int \frac{e^{\text{Tan}^{-1}x}}{1+x^2}\,dx$ on $I \subset (0,\infty)$

◆B. $\int \frac{\sin(\text{Tan}^{-1}x)}{1+x^2}\,dx$, $x \in \mathbb{R}$

◆C. $\int \frac{2x^3}{1+x^8}dx$ on R

◆D. $\int \frac{x^8}{1+x^{18}}dx$ on $\mathbb{R}$

◆E. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx$ on $(0,\infty)$

◆F. $\int \frac{x^2+1}{x^4+1}dx$ on $\mathbb{R}$

◆G. $\int \sec x \log(\sec x + \tan x)dx$ on $\left(0,\frac{\pi}{2}\right)$

◆H. $\int \sin^3 x\ dx$ on $\mathbb{R}$

◆I. $\int \cos^3 x\ dx$ on $\mathbb{R}$

◆J. $\int \cos x\ \cos 3x\ dx$ on $\mathbb{R}$

◆K. $\int \frac{dx}{1+e^x}$, $x \in \mathbb{R}$

◆L. $\int (\tan x + \log\sec x)e^x dx$ on $\left(\left(2n-\frac{1}{2}\right)\pi, \left(2n+\frac{1}{2}\right)\pi\right)$, $n \in \mathbb{Z}$

M. Evaluate $\int \sqrt{16-25x^2}\ dx$ on $\left(-\frac{4}{5},\frac{4}{5}\right)$

N. Evaluate $\int \frac{dx}{\sqrt{x^2+2x+10}}$

O. $\int \frac{1}{x\log x[\log(\log x)]}dx$ on $(1,\infty)$

P. $\int \cos x\ \cos 2x\ dx$ on $\mathbb{R}$

Q. $\int \sqrt{x}\log x\ dx$ on $(0,\infty)$

R. $\int e^x\left(\frac{1+x\log x}{x}\right)dx$ on $(0,\infty)$

S. $\int \frac{x\,e^x}{(x+1)^2}dx$ on $I \subset \mathbb{R} \setminus \{-1\}$

T. $\int e^x \frac{(x+2)}{(x+3)^2}dx$ on $I \subset \mathbb{R} \setminus \{-3\}$

U. $\int \frac{1}{1-\cot x}dx$

V. $\int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x}dx$ on $I \subset \mathbb{R} \setminus \left\{x \in \mathbb{R} \mid a\cos^2 x + b\sin^2 x \neq 0\right\}$

W. $\int e^x(\sec x + \sec x\tan x)dx$ on $I \subset \mathbb{R} \setminus \left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$

8.

Evaluate the following definite integrals

◆A. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{5}{2}} x}{\sin^{\frac{5}{2}} x + \cos^{\frac{5}{2}} x}dx$

◆B. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

◆C. $\int_0^{\pi} \sqrt{2+2\cos\theta}\ d\theta$

◆D. $\int_0^2 |1-x|dx$

◆E. $\int_0^{\frac{\pi}{1}} \frac{dx}{\sqrt{3-2x}}$

◆F. $\int_0^1 x\,e^{-x^2}dx$

◆G. $\int_1^5 \frac{dx}{\sqrt{2x-1}}$ H. $\int_2^3 \frac{2x}{1+x^2}\,dx$ I. $\int_0^{\frac{\pi}{2}} \sec^4\theta\, d\theta$ J. $\int_0^4 |2-x|\,dx$

K. Evaluate $\lim_{n\to\infty} \frac{2^k+4^k+6^k+.....+(2n)^k}{n^{k+1}}$ by using the method of finding definite integral as the limit of a sum.

9.

◆A. Find $\int_0^{\frac{\pi}{2}} \sin^4 x\,dx$

◆B. Find $\int_0^{\frac{\pi}{2}} \cos^8 x\,dx$

◆C. Find the area under the curve $f(x)=\sin x$ in $[0,2\pi]$.

◆D. Find the area under the curve $f(x)=\cos x$ in $[0,2\pi]$

◆E. Find the area bounded by the curves $y=\sin x$ and $y=\cos x$ between any two consecutive points of intersection.

◆F. Find the area enclosed within the curve $|x|+|y|=1$

◆G. Evaluate the following definite integrals. (i) $\int_0^{\frac{\pi}{2}} \sin^5 x\cos^4 x\,dx$ (ii) $\int_0^{\frac{\pi}{2}} \sin^6 x\cos^4 x\,dx$

H. Find $\int_0^{\frac{\pi}{2}} \sin^7 x\,dx$

I. Evaluate $\int_0^a \sqrt{a^2-x^2}\,dx$

J. Find the area bounded by the parabola $y=x^2$, the X-axis and the lines $x=-1, x=2$

K. Find the area bounded between the curves $y=x^2, y=\sqrt{x}$

L. Evaluate $\int_0^{\frac{\pi}{2}} \tan^5 x\cos^8 x\,dx$

10.

◆A. Find the order and degree of $\left(\frac{d^3y}{dx^3}\right)^2 - 3\left(\frac{dy}{dx}\right)^2 - e^x = 4$

◆B. Find the order and degree of $\left(\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3\right)^{6/5} = 6y$

◆C. Find the order of the differential equation corresponding to $y = Ae^{x} + Be^{3x} + Ce^{5x}$ (A, B, C *being parameters*) is a solution

◆D. Form the differential equation corresponding to $y = A\cos 3x + b\sin 3x$, where A and B are parameters.

◆E. Form the differential equation corresponding to the family of circles of radius r given by $(x-a)^2 + (y-b)^2 = r^2$, where a and b are parameters.

◆F. Solve $\frac{dy}{dx} + y\tan x = \sin x$

◆G. Solve $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$

H. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} = -p^2 y$.

I. Find the order of the differential equation corresponding to $y = c(x-c)^2$, where c is an arbitrary constant.

J. Find the order and degree of $x^{1/2}\left(\frac{d^2y}{dx^2}\right)^{1/3} + x\frac{dy}{dx} + y = 0$

K. Find the general solution of $x + y\frac{dy}{dx} = 0$

L. Find the general solution of $\frac{dy}{dx} = e^{x+y}$

M. Solve $y(1+x)dx + x(1+y)dy = 0$

N. Form the differential equations of the following family of curves where parameters are given in brackets.

(i) $xy = ae^{x} + be^{-x}$; (a,b) (ii) $y = (a+bx)e^{kx}$; (a,b) (iii) $y = a\cos(nx+b)$; (a,b)

O. Find the general solution of $\sqrt{1-x^2}\ dy + \sqrt{1-y^2}\ dx = 0$

SHORT ANSWER QUESTIONS

11.

◆A. From the point $A(0,3)$ on the circle $x^2 + 4x + (y-3)^2 = 0$ a chord AB is drawn and extended to a point M such that $AM = 2AB$. Find the equation of the locus of M.

◆B. If the abscissae of points A, B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and ordinates of A, B are roots of $y^2 + 2py - q^2 = 0$ then find the equation of a circle for which $\overline{AB}$ is a diameter.

◆C. Find the equation of a circle which is concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$ and passing through $(-2, 14)$.

◆D. If a point P is moving such that the length of tangents drawn from P to $x^2 + y^2 - 2x + 4y - 20 = 0$ and $x^2 + y^2 - 2x - 8y + 1 = 0$ are in the ratio 2:1 then show that the equation of the locus of P is $x^2 + y^2 - 2x - 12y + 8 = 0$.

◆E. If $4x-3y+7=0$ is a tangent to the circle represented by $x^2+y^2-6x+4y-12=0$ the find its point of contact.

◆F. Find the direct common tangents of the circles $x^2+y^2+22x-4y-100=0$ and $x^2+y^2-22x+4y+100=0$.

◆G. Find the transverse common tangents of the circles $x^2+y^2-4x-10y+28=0$ and $x^2+y^2+4x-6y+4=0$.

H. If a point P is moving such that the lengths of tangents drawn from P to the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$ are in the ratio 2 : 3 then find the equation of the locus of P.

I. Find the equations of the tangents to the circle $x^2+y^2-4x+6y-12=0$ which are parallel to $x+y-8=0$.

J. Find the equations of the tangents to the circle $x^2+y^2+2x-2y-3=0$ which are perpendicular to $3x-y+4=0$.

K. Find the equation of circles passing through $(1,-1)$, touching the lines $4x+3y+5=0$ and $3x-4y-10=0$.

L. Show that $x+y+1=0$ touches the circle $x^2+y^2-3x+7y+14=0$ and find its point of contact.

M. Tangents are drawn to the circle $x^2+y^2=16$ from the point $P(3,5)$. Find the area of the triangle formed by these tangents and the chord of contact of P.

N. Show that the circles $x^2+y^2-6x-2y+1=0, x^2+y^2+2x-8y+13=0$ touch each other

Find the point of contact and the equation of common tangent at their point of contact.

O. Show that $x^2+y^2-6x-9y+13=0, x^2+y^2-2x-16y=0$ touch each other. Find the point of contact and the equation of common tangent at their point of contact.

12.

◆A. Find the equation and length of the common chord of the two circles $S \equiv x^2+y^2+3x+5y+4=0$ and $S' \equiv x^2+y^2+5x+3y+4=0$.

◆B. If the two circles $x^2+y^2+2gx+2fy=0$ and $x^2+y^2+2g'x+2f'y=0$ touch each other then show that $f'g=fg'$.

◆C. Show that the circles $x^2+y^2+2ax+c=0$ and $x^2+y^2+2by+c=0$ touch each other if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c}$.

◆D. Show that the circles $x^2+y^2-2x=0$ and $x^2+y^2+6x-6y+2=0$ touch each other. Find the coordinates of the point of contact. Is the point of contact external or internal?

E. Show that the circles $x^2+y^2-4x-6y-12=0$ and $5(x^2+y^2)-8x-14y-32=0$ touch each other and find their point of contact.

F. Show that four common tangents can be drawn for the circles given by $x^2+y^2-14x+6y+33=0$ and $x^2+y^2+30x-2y+1=0$ and find the internal and external centres of similitude.

13.

◆A. Find the eccentricity, coordinates of foci, Length of latus rectum and equations of directrices of the following ellipse.

(i) $9x^2+16y^2-36x+32y-92=0$ (ii) $3x^2+y^2-6x-2y-5=0$

◆B. Find the equation of the ellipse referred to its major and minor axes as the coordinate axes X, Y-respectively with latus rectum of length 4 and distance between foci $4\sqrt{2}$.

◆C. If θ_1,θ_2 are the eccentric angles of the extremities of a focal chord (other than the vertices) of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $(a>b)$ and e its eccentricity. Then show that

(i) $e\cos\frac{(\theta_1+\theta_2)}{2}=\cos\frac{(\theta_1-\theta_2)}{2}$ (ii) $\frac{e+1}{e-1}=\cot\left(\frac{\theta_1}{2}\right)\cot\left(\frac{\theta_2}{2}\right)$

◆D. Find the equation of the ellipse with focus at $(1,-1)$, $e=2/3$ and directrix as $x+y+2=0$.

◆E. Prove that the equation of the chord joining the points 'α' and 'β' on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$

F. S and T are the foci of an ellipse and B is one end of the minor axis. If STB is an equilateral triangle, then fund the eccentricity of the ellipse.

G. Find the equation of the ellipse in the standard form such that distance between foci is 8 and distance between directrices is 32.

H. Find the length of major axis, minor axis, latus rectum, eccentricity, coordinates of centre, foci and the equations of directrices of the following ellipse.

(i) $9x^2+16y^2=144$ (ii) $x^2+2y^2-4x+12y+14=0$

I. A man running on a race course notices that the sum of the distances of the two flag posts from him is always 10m. and the distance between the flag posts is 8m. Find the equation of the race course traced by the man.

14.

◆A. Find the equation of tangent and normal to the ellipse $9x^2+16y^2=144$ at the end of the latus rectum in the first quadrant.

◆B. If the normal at one end of a latusrectum of the of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through one end of the minor axis, then show that $e^4+e^2=1$ (e is the eccentricity of the ellipse)

◆C. Find the equation of the tangents to the ellipse $2x^2+y^2=8$ which are

(i) parallel to $x-2y-4=0$ (ii) perpendicular to $x+y+2=0$

(iii) which makes an angle $\frac{\pi}{4}$ with x-axis

◆D. A circle of radius 4, is concentric with the ellipse $3x^2+13y^2=78$. Prove that a common tangent is inclined to the major axis at an angle $\frac{\pi}{4}$.

◆E. Show that the locus of the feet of the perpendiculars drawn from foci to any tangent of the ellipse is the auxiliary circle.

F. If a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\ (a>b)$ meets its major axis and minor axis at M and N respectively then prove that $\frac{a^2}{(CM)^2}+\frac{b^2}{(CN)^2}=1$ Where C is the centre of the ellipse.

G. If PN is the ordinate of a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the tangent at P meets the X-axis at T then show that $(CN)(CT)=a^2$ where C is the centre of the ellipse.

H. Show that the points of intersection of the perpendicular tangents to an ellipse lie on a circle.

I. Find the equation of tangent and normal to the ellipse $x^2+8y^2=33$ at $(-1,2)$.

J. Find the condition for the line $x\cos\alpha+y\sin\alpha=p$ to be a tangent to the ellipse ...

K. Show that the foot of the perpendicular drawn from the centre on any tangent to the ellipse lies on the curve $\left(x^2+y^2\right)^2=a^2x^2+b^2y^2$.

15.

◆A. Find the centre, eccentricity, foci, directrices and the length of the latus rectum of the following hyperbolas.

(i) $4x^2-9y^2-8x-32=0$ (ii) $4(y+3)^2-9(x-2)^2=1$

◆B. Prove that the point of intersection of two perpendicular tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ lies on the circle $x^2+y^2=a^2-b^2$.

◆C. Find the centre, foci, eccentricity, equation of the directrices, length of the latus rectum of the following hyperbolas

(i) $x^2-4y^2=4$ (ii) $9x^2-16y^2+72x-32y-16=0$

◆D. Show that the angle between the two asymptotes of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $2\,\mathrm{Tan}^{-1}\left(\frac{b}{a}\right)$ or $2\sec^{-1}(e)$.

◆E. Find the equations of the tangents to the hyperbola $x^2-4y^2=4$ which are (i) parallel (ii) perpendicular to the line $x+2y=0$.

F. Find the equation of tangents drawn to the hyperbola $2x^2-3y^2=6$ through (–2, 1)

G. If the line $lx+my+n=0$ is a tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ then, show that $a^2l^2-b^2m^2=n^2$

H. Prove that the product of the perpendicular distances from any point on a hyperbola to its asymptotes is constant.

I. Tangents to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ make angles θ_1, θ_2 with transverse axis of a hyperbola. Show that the point of intersection of these tangents lies on the curve $2xy = k(x^2 - a^2)$ when $\tan\theta_1 + \tan\theta_2 = k$.

J. Show that the locus of feet of the perpendiculars drawn from foci to any tangent of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is the auxiliary circle of the hyperbola.

16.

◆A. Evaluate $\lim_{n\to\infty}\left[\left(1+\frac{2}{n}\right)......\left(1+\frac{n}{n}\right)\right]^{\frac{1}{n}}$

◆B. Evaluate $\lim_{n\to\infty}\frac{\sqrt{n+1}+\sqrt{n+2}+.....+\sqrt{n+n}}{n\sqrt{n}}$

◆ C. Find the area of one of the curvilinear triangles bounded by $y=\sin x$, $y=\cos x$ and X-axis.

D. Find $\int_{-a}^{a} x^2\left(a^2-x^2\right)^{\frac{3}{2}} dx$

E. Find $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos^4 x \, dx$

F. Find the area enclosed by the curves $y=3x$ and $y=6x-x^2$

G. Evaluate $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{6n}\right]$

H. Evaluate $\lim_{n\to\infty}\frac{1}{n}\left[\tan\frac{\Pi}{4n}+\tan\frac{2\Pi}{4n}+......+\tan\frac{n\Pi}{4n}\right]$

I. Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^3}{i^4+n^4}$

J. Evaluate $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)......\left(1+\frac{n^2}{n^2}\right)\right]^{\frac{1}{n}}$

K. Evaluate $\int_{0}^{\frac{\pi}{2}}\frac{dx}{4+5\cos x}$

L. Evaluate $\int_{0}^{1} x Tan^{-1}x \, dx$

M. Find the area of the region bounced by the parabolas $y^2=4x$ and $x^2=4y$

17.

Solve the following differential equations:

◆A. $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$

◆B. $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$

◆C. $\frac{dy}{dx}-x\tan(y-x)=1$

◆D. $(x^2+y^2)dx=2xydy$

◆E. $(\cos x)\frac{dy}{dx}+y\sin x=\tan x$

◆F. $\left(1+x^2\right)\frac{dy}{dx}+2xy-4x^2=0$

◆G. $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$

◆H. $\left(1+x^2\right)\frac{dy}{dx}+y=e^{Tan^{-1}x}$

◆I. $\frac{dy}{dx}+\frac{4x}{1+x^2}y=\frac{1}{\left(1+x^2\right)^2}$

◆J. $\cos x.\frac{dy}{dx}+y\sin x=\sec^2 x$

◆K. $\left(x+2y^3\right)\frac{dy}{dx}=y$

◆L. $\frac{dy}{dx}\left(x^2y^3+xy\right)=1$

◆M. $xdy=\left(y+x\cos^2\frac{y}{x}\right)dx$

◆N. $\left(x^2-y^2\right)\frac{dy}{dx}=xy$

O. $\frac{dy}{dx}=\frac{xy+y}{xy+x}$

P. $\left(xy^2+x\right)dx+\left(yx^2+y\right)dy=0$

Q. $\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$

R. $\frac{dy}{dx}=\tan^2(x+y)$

S. $\left(1+e^{x/y}\right)dx+e^{x/y}\left(1-\frac{x}{y}\right)dy=0$

T. $(x+y+1)\frac{dy}{dx}=1$

U. $\frac{dy}{dx}=\sin(x+y)+\cos(x+y)$

V. $\frac{dy}{dx}=\frac{x-2y+1}{2x-4y}$

LONG ANSWER QUESTIONS

18.

◆A. Find the equation of a circle which passes through $(2,-3)$ and $(-4,5)$ and having the centre on $4x+3y+1=0$.

◆B. Find the equation of a circle which passes through (4, 1), (6, 5) and having the centre on $4x+3y-24=0$.

◆C. Find the equation of the circle whose centre lies on the X-axis and passing through $(-2,3)$ and $(4,5)$.

◆D. Find the equation of circle passing through the points $(1,2),(3,-4),(5,-6)$

◆E. Find the equation of the circle passing through (0, 0) and making intercepts 4, 3 on X-axis and Y-axis is respectively.

◆F. Find the equation of the circle passing through (0, 0) and making intercept 6 units on X-axis and intercept 4 units on Y-axis.

◆G. If $(2,0),(0,1)(4,5)$ and $(0,c)$ are concylclic then find c.

◆H. Find the equations of circles which touch $2x-3y+1=0$ at (1,1) and having radius $\sqrt{13}$.

◆I. Show that the circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x+18y+26=0$ touch each other. Also find the point of contact and common tangent at this point of contact.

◆J. Show that the tangent at $(-1,2)$ of the circle $x^2+y^2-4x-8y+7=0$ touches the circle $x^2+y^2+4x+6y=0$ and also find its point of contact.

◆K. If the polar of the points on the circle $x^2+y^2=a^2$ with respect to the circle $x^2+y^2=b^2$ touches the circle $x^2+y^2=c^2$ then prove that a,b,c are in Geometrical progression.

L. If θ_1,θ_2 are the angles of inclination of tangents through a point P to the circle $x^2+y^2=a^2$ then find the locus of P when $\cot\theta_1+\cot\theta_2=k$.

M. Prove that the tangent at $(3,-2)$ of the circle $x^2+y^2=13$ touches the circle $x^2+y^2+2x-10y-26=0$ and find its point of contact.

N. Find the equation of the circle which touches the circle $x^2+y^2-2x-4y-20=0$ externally at $(5,5)$ with radius 5.

O. Find the pair of tangents drawn from $(1,3)$ to the circle $x^2+y^2-2x+4y-11=0$ and also find the angle between them.

19.

◆A. Find the equation of the circle which passes through (1, 1) and cuts orthogonally each of the circles $x^2+y^2-8x-2y+16=0$ and $x^2+y^2-4x-4y-1=0$.

◆B. Find the equation of the circle which intersects the circle $x^2+y^2-6x+4y-3=0$ orthogonally and passes through the point (3, 0) and touches Y-axis.

C. Find the equations to all possible common tangents of the circles $x^2+y^2-2x-6y+6=0$ and $x^2+y^2=1$

D. Find all common tangents of the following pairs of circles.

(i) $x^2+y^2=9$ and $x^2+y^2-16x+2y+49=0$

(ii) $x^2+y^2+4x+2y-4=0$ and $x^2+y^2-4x-2y+4=0$

E. Find the equation of the circle which is orthogonal to each of the following three circles $x^2+y^2+2x+17y+4=0$, $x^2+y^2+7x+6y+11=0$ and $x^2+y^2-x+22y+3=0$.

F. Find the equation of the circle passing through the origin, having its centre on the line $x+y=4$ and intersecting the circle $x^2+y^2-4x+2y+4=0$ orthogonally.

G. Find the equation of the circle which cuts the circles $x^2+y^2-4x-6y+11=0$ and $x^2+y^2-10x-4y+21=0$ orthogonally and has the diameter along the straight line $2x+3y=7$.

H. If $x+y=3$ is the equation of the chord AB of the circle $x^2+y^2-2x+4y-8=0$, find the equation of the circle having $\overline{AB}$ as diameter.

I. Show that the common chord of the circles $x^2+y^2-6x-4y+9=0$ and $x^2+y^2-8x-6y+23=0$ is the diameter of the second circle and also find its length.

20.

◆A. Show that the common tangent to the parabola $y^2 = 4ax$ and $x^2 = 4by$ is $xa^{1/3} + yb^{1/3} + a^{2/3}b^{2/3} = 0$

◆B. Prove that area of the triangle formed by the tangents at $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) to the parabola $y^2 = 4ax \ (a > 0)$ is $\frac{1}{16a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$ sq. units.

◆C. Find the equation of the parabola whose axis is parallel to x-axis and which passes through the points $(-2,1)$, $(1,2)$ and $(-1,3)$.

◆D. Find the equation of the parabola whose axis is parallel to y-axis and which passes through the points $(4,5)$, $(-2,11)$ and $(-4,21)$.

◆E. Prove that the area of the triangle inscribed in the parabola $y^2 = 4ax$ is $\frac{1}{8a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right|$ sq. units where y_1, y_2, y_3 are the ordinates of its vertices.

◆F. Find the equations of tangents to the parabola $y^2 = 16x$ which are parallel and perpendicular respectively to the line $2x - y + 5 = 0$, also find the coordinates of their points of contact.

◆G. Show that the equation of common tangent to the circle $x^2 + y^2 = 2a^2$ and the parabola $y^2 = 8ax$ are $y = \pm(x + 2a)$.

◆H. The normal at a point t_1 on $y^2 = 4ax$ meets the parabola again in the point t_2. Then prove that $t_1t_2 + t_1^2 + 2 = 0$.

◆I. If a normal chord a point 't' on the parabola $y^2 = 4ax$ subtends a right angle at vertex, then prove that $t = \pm\sqrt{2}$.

J. Find the coordinates of the vertex and focus, and the equations of the directrix and axes of the following parabolas

(i) $y^2 = 16x$ (ii) $x^2 = -4y$ ◆(iii) $3x^2 - 9x + 5y - 2 = 0$ ◆(iv) $y^2 - x + 4y + 5 = 0$

K. From an external point P, tangent are drawn to the parabola $y^2 = 4ax$ and these tangents make angles θ_1, θ_2 with its axis, such that $\tan\theta_1 + \tan\theta_2$ is a constant b. Then show that P lies on the line $y = bx$.

L. Prove that the two parabolas $y^2 = 4ax$ and $x^2 = 4by$ intersect (other than the origin) at an angle of $Tan^{-1}\left[\frac{3a^{1/3}b^{1/3}}{2\left(a^{2/3} + b^{2/3}\right)}\right]$.

M. Find the equation of the parabola whose focus is $(-2,3)$ and directrix is the line $2x + 3y - 4 = 0$. Also find the length of the latus rectum and the equation of the axis of the parabola.

N. Find the coordinates of the vertex and focus, the equation of the directrix and axis of the following parabolas.

(i) $y^2 + 4x + 4y - 3 = 0$ (ii) $x^2 - 2x + 4y - 3 = 0$

O. Show that the locus of point of intersection of perpendicular tangents to the parabola $y^2 = 4ax$ is the directrix $x + a = 0$.

P. Show that the common tangents to the circle $2x^2 + 2y^2 = a^2$ and the parabola $y^2 = 4ax$ intersect at the focus of the parabola $y^2 = -4ax$.

21.

Evaluate

◆A. $\int (3x-2)\sqrt{2x^2 - x + 1}\,dx$

◆B. $\int \frac{2x+5}{\sqrt{x^2 - 2x + 10}}dx$

◆C. $\int (6x+5)\sqrt{6 - 2x^2 + x}\,dx$

◆D. $\int \frac{dx}{(1+x)\sqrt{3 + 2x - x^2}}$ on $(-1,3)$

◆E. $\int \frac{2x+3}{(x+3)\left(x^2+4\right)}dx$

F. $\int \frac{dx}{(x+1)\sqrt{2x^2 + 3x + 1}}$ on $I \subset \mathbb{R} \setminus \left[-1, -\frac{1}{2}\right]$

◆G. $\int \frac{\cos x + 3\sin x + 7}{\cos x + \sin x + 1}dx$

◆H. $\int \frac{2\cos x + 3\sin x}{4\cos x + 5\sin x}dx$

◆I. $\int \frac{9\cos x - \sin x}{4\sin x + 5\cos x}dx$

◆J. $\int \frac{1}{1 + \sin x + \cos x}dx$

◆K. $\int \frac{2\sin x + 3\cos x + 4}{3\sin x + 4\cos x + 5}dx$

L. $\int \frac{1}{(x-a)(x-b)(x-c)}dx$

M. $\int \frac{dx}{1+x^4}$ on $\mathbb{R}$

N. $\int \frac{x+1}{\sqrt{x^2 - x + 1}}\,dx$

O. $\int \frac{dx}{4 + 5\sin x}$

P. $\int \frac{1}{2 - 3\cos 2x}dx$

Q. $\int \frac{dx}{4\cos x + 3\sin x}$

R. $\int \frac{dx}{(1-x)\sqrt{3 - 2x - x^2}}$ on $(-3,1)$

S. $\int \frac{x+3}{(x-1)\left(x^2+1\right)}dx$

T. $\int \frac{x^3 - 2x + 3}{x^2 + x - 2}dx$

U. $\int \frac{2x^2 + x + 1}{(x+3)(x-2)^2}dx$

V. $\int \frac{dx}{x^3 + 1}$

W. $\int \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2}dx$

X. $\int \frac{dx}{3\cos x + 4\sin x + 6}$

22.

◆A. If $I_n = \int \cos^n x\,dx$, then show that $I_n = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n} I_{n-2}$, n being a positive integer, $n \ge 2$ and deduce the value of $\int \cos^4 x dx$.

◆B. Obtain reduction formula for $I_n = \int \cot^n x dx$, n being a positive integer, $n \ge 2$ and deduce the value of $\int \cot^4 x dx$.

◆C. Obtain the reduction formula for $I_n = \int \operatorname{cosec}^n x\, dx$, n being a positive integer, $n \geq 2$ and deduce the value of $\int \operatorname{cosec}^5 x\, dx$.

D. If $I_{m,n} = \int \sin^m x \cos^n x\, dx$, then show that

$I_{m,n} = -\dfrac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \dfrac{m-1}{m+n} I_{m-2,n}$, for a positive integer n and an integer $m \geq 2$.

◆E. Obtain reduction formula for $\int \sin^n x$ for an integer $n \geq 2$ and deduce the value of $\int \sin^4 x\, dx$.

◆F. Obtain reduction formula for $\int \tan^n x$ $I_n = \int \cot^n x dx$, n being a positive integer, $n \geq 2$ and deduce the value of $\int \tan^6 x\, dx$.

◆G. Obtain reduction formula for $\int \sec^n x$ for an integer $n \geq 2$ and deduce the value of $\int \sec^5 x\, dx$.

23.

◆A. Show that the area enclosed between the curves $y^2 = 12(x+3)$ and $y^2 = 20(5-x)$ is $64\sqrt{\dfrac{5}{3}}$.

◆B. Show that the area of the region bounced by $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ (ellipse) is πab. Also deduce the area of the circle $x^2 + y^2 = a^2$.

◆C. Find the area bounded between the curves $y^2 = 4ax$, $x^2 = 4by$ $(a > 0, b > 0)$

◆D. Find the area bounded between the curves

(i) $y = 4x - x^2$ and $y = 5 - 2x$ (ii) $y^2 = 4x$, $y^2 = 4(4-x)$

Evaluate

◆E. $\displaystyle\int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$ ◆F. $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$ ◆G. $\displaystyle\int_0^{1} \frac{\log(1+x)}{1+x^2} dx$

◆H. $\displaystyle\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx$ ◆I. $\displaystyle\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ J. $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx$

K. $\displaystyle\int_a^{b} \sqrt{(x-a)(b-x)}\, dx$ L. $\displaystyle\int_0^{\pi} \frac{x \sin^3 x}{1+\cos^2 x} dx$ M. $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx$

N. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\cos x + \sin x}\,dx$

O. The circle $x^2 + y^2 = 8$ is divided into two parts by parabola $2y = x^2$. Find the area of both the parts.

P. Let AOB be the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $OA = a$, $OB = b$. Then show that the area bounded between the chord AB and the arc AB of the ellipse is $\frac{(\pi - 2)ab}{4}$.

24.

◆A. Find the equation of a curve whose gradients is $\frac{dy}{dx} = \frac{y}{x} - \cos^2 \frac{y}{x}$, where $x > 0, y > 0$ and which passes through the point $\left(1, \frac{\pi}{4}\right)$.

◆B. Solve $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$

◆C. Solve $(2x + y + 1)dx + (4x + 2y - 1)dy = 0$

◆D. Solve $\frac{dy}{dx} = \frac{3y - 7x + 7}{3x - 7y - 3}$

◆E. Solve $\frac{dy}{dx} + y\tan x = \sin x$

◆F. Solve $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$

G. Solve $\frac{dy}{dx} = \frac{x^2 + y^2}{2x^2}$

H. Solve $x\sec\left(\frac{y}{x}\right).(ydx + xdy) = y\text{cosec}\left(\frac{y}{x}\right).(xdy - ydx)$

I. Give the solution of $x\sin^2 \frac{y}{x} dx = ydx - xdy$ which passes through the point $\left(1, \frac{\pi}{4}\right)$

J. Find the solution of the equation $x(x-2)\frac{dy}{dx} - 2(x-1)y = x^3(x-2)$ which satisfies the condition that $y = 9$ when $x = 3$.

K. Solve $\left(y^2 - 2xy\right)dx + \left(2xy - x^2\right)dy = 0$

L. Solve: $(y\,dx + x\,dy)x\cos\frac{y}{x} = (x\,dy - y\,dx)y\sin\frac{y}{x}$

M. Solve $(2x + 2y + 3)\frac{dy}{dx} = x + y + 1$

N. Solve $\left(1 + x^2\right)\frac{dy}{dx} + y = Tan^{-1}x$

O. Solve $(x - y)dy = (x + y + 1)dx$

P. Solve $\left(x^2 + y^2\right)dx = 2xy\,dy$

🌹wish you all the best🌹