FIITJEE

KUKATPALLY CENTRE

IMPORTANT QUESTIONS

FOR

INTERMEDIATE PUBLIC EXAMINATIONS

IN

MATHS-IIA

2017-18

INTERMEDIATE PUBLIC EXAMINATION, MARCH 2017

Total No. of Questions - 24 **Reg. No.**

Total No. of Printed Pages - 2

Part - III
MATHEMATICS, Paper-II (A)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

SECTION - A $10 \times 2 = 20$ M

I. Very Short Answer Type questions:

1. Write the complex number $(1+2i)^3$ in the form of $a+ib$.
2. Express the complex number $1+i\sqrt{3}$ in modulus amplitude form.
3. If $1,\omega,\omega^2$ are the cube roots of unity, then prove that $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})=49$.
4. If $x^2-6x+5=0$ and $x^2-12x+p=0$ have a common root, then find p.
5. Form the polynomial equation of the lowest degree with roots as $0,0,2,2,-2,-2$.
6. If ${}^nP_7=42\cdot\left({}^nP_5\right)$, then find n.
7. Find the value of ${}^{10}C_5+2\cdot\left({}^{10}C_4\right)+{}^{10}C_3$.
8. Find the set E of the values of x for which the binomial expansion $(2+5x)^{\frac{-1}{2}}$ is valid.
9. Find the mean deviation about the mean for the following data: 3, 6, 10, 4, 9, 10
10. The mean and variance of a binomial distribution are 4 and 3 respectively. Fix the distribution and find $P(X\geq 1)$.

SECTION – B $5 \times 4 = 20$ M

II. Short Answer Type questions:

(i) Attempt any **five** questions

(ii) Each question carries **four** marks

11. If $x+iy=\dfrac{3}{2+\cos\theta+i\sin\theta}$ then, show that $x^2+y^2=4x-3$.
12. If x_1,x_2 are the roots of the quadratic equation $ax^2+bx+c=0$ and $c\neq 0$, find the value of $(ax_1+b)^{-2}+(ax_2+b)^{-2}$ in terms of a,b,c.
13. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order, find the rank of the word EAMCET.
14. Prove that, $\dfrac{{}^{4n}C_{2n}}{{}^{2n}C_n}=\dfrac{1\cdot3\cdot5.....(4n-1)}{\left[1\cdot3\cdot5.......(2n-1)\right]^2}$.
15. Resolve the fraction $\dfrac{2x^2+2x+1}{x^3+x^2}$ into partial fraction.

16. A and B are events with $P(A)=0.5$, $P(B)=0.4$ and $P(A\cap B)=0.3$. Find the probability that

(i) A does not occur (ii) neither A nor B occurs.

17. Find the probability of drawing an Ace or a Spade from a well shuffled pack of 52 playing cards.

SECTION – C **5 × 7 = 35 M**

III. Long Answer Type questions:

(i) Attempt any **five** questions

(ii) Each question carries **seven** marks

18. If n is an integer then show that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$.

19. Show that $x^5-5x^3+5x^2-1=0$, has three equal roots and find that root.

20. If n is a positive integer, then prove that $C_0+\frac{C_1}{2}+\frac{C_2}{3}+......+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$

21. Find the sum of the series $\frac{3\cdot 5}{5\cdot 10}+\frac{3\cdot 5\cdot 7}{5\cdot 10\cdot 15}+\frac{3\cdot 5\cdot 7\cdot 9}{5\cdot 10\cdot 15\cdot 20}+......\infty$

22. Calculate the variance and standard deviation of the following continuous frequency distribution:

Class interval	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 - 90	90 – 100
Frequency	3	7	12	15	8	3	2

23. State and prove Baye's theorem.

24.

$X=x$	-2	-1	0	1	2	3
$P(X=x)$	0.1	K	0.2	$2K$	0.3	K

Is the probability distribution of a random variable X. Find the value of K and the variance of X.

BLUE PRINT (MATHS-IIA)

S.No.	Chapter Name	Weightage Marks
ALGEBRA		
1.	**Complex Numbers**	**8 (2 + 2 + 4)**
2.	**Demovier's Theorem**	**9 (2 + 7)**
3.	**Quadratic Equations**	**15 (2 + 2 + 4 + 7)**
4.	**Permutations and Combinations**	**12 (2 + 2 + 4 + 4)**
5.	**Probability**	**15 (4 + 4 + 7)**
6.	**Binomial Theorem**	**16 (2 + 7 + 7)**
7.	**Partial Fractions**	**4**
8.	**Measures of Dispersion**	**9 (2 + 7)**
9.	**Random Variables**	**16 (2 + 7)**

VERY SHORT ANSWER QUESTIONS

1.

◆A. Show that $\frac{2-i}{(1-2i)^2}$ and $\frac{-2-11i}{25}$ are conjugate to each other.

◆B. Find the multiplicative inverse of $7+24i$.

◆C. Show that $z_1=\frac{2+11i}{25}, z_2=\frac{-2+i}{(1-2i)^2}$ are conjugate to each other.

◆D. Find the square roots of $(-5+12i)$.

◆E. If $z_1=(6,3); z_2=(2,-1)$ find z_1 / z_2.

◆F. If $z=(\cos\theta,\sin\theta)$, find $\left(z-\frac{1}{z}\right)$.

G. Write the conjugate of the following complex numbers

◆(i) $(2+5i)(-4+6i)$ (Ii) $(15+3i)-(4-20i)$

H. Simplify ◆ (i) $i^2+i^4+i^6+......+(2n+1)$ terms (ii) $i^8-3.i^7+i^2\left(1+i^4\right)(-i)^{26}$

I. Find the real and imaginary parts of the complex number $\frac{a+ib}{a-ib}$.

J. Express $(1-i)^3(1+i)$ in the form of $a+ib$.

K. Write $z=-\sqrt{7}+i\sqrt{21}$ in the polar form.

L. Express $-1-i$ in polar form with principal value of the amplitude.

M. If $z_1=(2,-1), z_2=(6,3)$ find z_1-z_2.

N. If $z_2=(3,5)$ and $z_2=(2,6)$ find z_1z_2.

O. Write the complex number $\frac{4+3i}{(2+3i)(4-3i)}$ in the form $A+iB$.

P. If $(a+ib)^2=x+iy$, find x^2+y^2.

2.

◆A. (i) If $z_1=-1$ and $z_2=-i$, then find $Arg(z_1z_2)$ (ii) If $z_1=-1$ and $z_2=i$, then find $Arg\left(\frac{z_1}{z_2}\right)$

◆B. If $(1-i)(2-i)(3-i).....(n-i)=x-iy$, then prove that $2.5.10.....(1+n)^2=x^2+y^2$

◆C. Show that the four points in the Argand plane represented by the complex numbers $2+i, 4+3i, 2+5i, 3i$ are the vertices of a square.

D. ◆ (i) If $z=\frac{1+2i}{1-(1-i)^2}$, then find $Arg(z)$.

(ii) If the amplitude of $(z-1)$ is $\frac{\pi}{2}$, then find the locus of z.

(iii) If the $Arg\bar{z}_1$ and $Argz_2$ are $\frac{\pi}{5}$ and $\frac{\pi}{3}$ respectively, then find $(Argz_1+Argz_2)$.

E. If $z = x + iy$ and if the point P in the Argand plane represents z, then describe geometrically the locus of z satisfying the equations.

◆(i) $|z-2-3i| = 5$ (ii) $2|z-2| = |z-1|$ (iii) Img $z^2 = 4$

F. ◆(i) If $\frac{z_2}{z_1}$, $z_1 \neq 0$, is an imaginary number then find the value of $\left|\frac{2z_1+z_2}{2z_1-z_2}\right|$.

◆(ii) If $\left(\sqrt{3}+i\right)^{100} = 2^{99}(a+ib)$, show that $a^2+b^2 = 4$.

(iii) If $\sqrt{3}+i = r(\cos\theta + i\sin\theta)$, then find the value of θ in radian measure.

(iv) If $x+iy = cis\alpha . cis\beta$, then find the value of x^2+y^2.

G. Show that the complex numbers z satisfying $z^2+\bar{z}^2 = 2$ constitute a hyperbola.

H. Show that the points in the Argand diagram represented by the complex numbers $1+3i, 4-3i, 5-5i$ are collinear.

I. Express the following complex numbers in modulus –amplitude form

(i) $1-i$ (ii) $1+i\sqrt{3}$ (iii) $-\sqrt{3}+i$ (iv) $-1-i\sqrt{3}$

J. Simplify the following complex numbers and find their modulus $\frac{(2+4i)(-1+2i)}{(-1-i)(3-i)}$

K. Find the equation of the perpendicular bisector of the line segment joining the points $7+7i$, $7-7i$ in the Argand plane.

L. Show that the points in the Argand diagram represented by the complex numbers $2+2i, -2-2i$, $-2\sqrt{3}+2\sqrt{3}i$ are the vertices of an equilateral triangle.

3.

◆A. Find all the values of

(i) $\left(1-i\sqrt{3}\right)^{1/3}$ (ii) $(-i)^{1/6}$ (iii) $(1+i)^{2/3}$ (iv) $(-16)^{1/4}$ (v) $(-32)^{1/5}$

◆B. (i) If $x = cis\theta$, then find the value of $\left(x^6+\frac{1}{x^6}\right)$ (ii) Find the cube roots of 8

C. Find the values of the following

◆(i) $\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5 - \left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$ (ii) $\left(1+i\sqrt{3}\right)^3$ (iii) $(1-i)^8$ (iv) $(1+i)^{16}$

D. If $1, \omega, \omega^2$ are the cube roots of unity, then prove that

◆ (i) $\frac{1}{2+\omega}+\frac{1}{1+2\omega} = \frac{1}{1+\omega}$

◆ (ii) $(2-\omega)\left(2-\omega^2\right)\left(2-\omega^{10}\right)\left(2-\omega^{10}\right)\left(2-\omega^{11}\right) = 49$

(iii) $(x+y+z)\left(x+y\omega+z\omega^2\right)\left(x+y\omega^2+z\omega\right) = x^3+y^3+z^3-3xyz$

E. If $1, \omega, \omega^2$ are the cube roots of unity, then find the values of the following.

◆ (i) $(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$ (ii) $(a+2b)^2+(a\omega^2+2b\omega)^2+(a\omega+2b\omega^2)^2$

(iii) $\left(\frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}\right)+\frac{(a+b\omega+c\omega^2)}{(b+c\omega+a\omega^2)}$ (iv) $(a+b)^3+(a\omega+b\omega^2)^3+(a\omega^2+b\omega)^3$

(v) $(1-\omega+\omega^2)^3$ (vi) $(1+\omega)^3+(1+\omega^2)^3$ (vii) $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$

◆F. If ABC are angles of a triangle such that $x = cis A, y = cis B, z = cis C$, then find the value of xyz.

◆G. Simplify $\frac{(\cos\alpha + i\sin\alpha)^4}{(\sin\beta + i\cos\beta)^8}$.

4.

◆A. For what values of m, the equation $x^2 - 2(1+3m)x + 7(3+2m) = 0$ will have equal roots?

◆B. Form a quadratic equation whose roots are $2\sqrt{3}-5$ and $-2\sqrt{3}-5$.

◆C. Solve $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 2 = 0$.

◆D. Find the values of m for the $(m+1)x^2 + 2(m+3)x + (m+8) = 0$ equation have equal roots?

◆E. If $x^2 + bx + c = 0$, $x^2 + cx + b = 0 (b \neq c)$ have a common root, then show that $b + c + 1 = 0$.

◆F. If the quadratic equations $ax^2 + 2bx + c = 0$ and $ax^2 + 2cx + b = 0, (b \neq 0)$ have a common root, then show that $a + 4b + 4c = 0$.

◆G. If $x^2 - 6x + 5 = 0$ and $x^2 - 3ax + 35 = 0$ have a common root, then find a.

◆H. Discuss the signs of the following quadratic expressions when x is real.

(i) $x^2 - 5x + 4$ (ii) $x^2 - x + 3$

◆I. Find the maximum or minimum of the following expressions as x varies over $\mathbb{R}$.

(i) $x^2 - x + 7$ (ii) $12x - x^2 - 32$ (iii) $2x + 5 - 3x^2$ (iv) $ax^2 + bx + a$ $(a, b, \in \mathbb{R}$ and $a \neq 0)$

◆J. If α, β are the roots of the equation $ax^2 + bx + c = 0$, find the values of the following expressions in terms of a, b, c.

(i) $\frac{\alpha^2+\beta^2}{\alpha^{-2}+\beta^{-2}}, c \neq 0$ (ii) $\frac{1}{\alpha}+\frac{1}{\beta}$ (iii) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ (iv) $\alpha^4\beta^7+\alpha^7\beta^4$ (v) $\left(\frac{\alpha}{\beta}-\frac{\beta}{\alpha}\right)^2$, if $c \neq 0$

K. Find the roots of the equation $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$.

L. Find the nature of the roots of $3x^2 + 7x + 2 = 0$

M. If α and β are the roots of $ax^2 + bx + c = 0$, find the values of $\alpha^2 + \beta^2$ and $\alpha^3 + \beta^3$ in terms of a, b, c.

N. For what values of x, the following expression are negative?

(i) $x^2 - 7x + 10$ (ii) $15 + 4x - 3x^2$ (iii) $2x^2 + 5x - 3$ (iv) $x^2 - 5x - 6$

5.

◆A. Form the monic polynomial equation of degree 3 whose roots are 2,3 and 6.

◆B. Form polynomial equations of the lowest degree, with roots $1,-1,3$

◆C. If $1,1,\alpha$ are the roots of $x^3-6x^2+9x-4=0$, then find α.

◆D. If $1,-2$ and 3 are the roots of $x^3-2x^2+ax+6=0$, then find a.

◆E. If the product of the roots of $4x^3+16x^2-9x-a=0$ is 9, then find a.

◆F. If α,β and 1 are the roots of $x^3-2x^2-5x+6=0$, then find α and β.

◆G. Find the polynomial equation whose roots are the reciprocals of the roots of $x^4-3x^3+7x^2+5x-2=0$.

H. Let α,β,γ be the roots of $x^3+px^2+qx+r=0$. Then find

◆(i) $\sum\alpha^3$ (ii) $\sum\alpha^2$ (iii) $\sum\frac{1}{\alpha}$, if α,β,γ are non-zero (iv) $\sum\beta^2\gamma^2$ (v) $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$

I. If $-1,2$ and α are the roots of $2x^3+x^2-7x-6=0$, then find α.

J. If α,β and γ are the roots of $x^3-2x^2+3x-4=0$, then find (i) $\Sigma\alpha^2\beta^2$ (ii) $\Sigma\alpha\beta(\alpha+\beta)$

K. Find the transformed equation whose roots are the negatives of the roots of $x^4+5x^3+11x+3=0$.

6.

◆A. If ${}^nP_4=1680$, find n.

◆B. Find the number of ways of arranging 6 boys and 6 girls in a row. In how many of these arrangements
i) all the girls sit together ii) no two girls sit together iii) boys and girls sit alternately

◆C. If ${}^nP_3=1320$, find n.

◆D. If ${}^nP_7=42.{}^nP_5$, find n.

◆E. If ${}^{(n+1)}P_5:{}^nP_6=2:7$, find n.

◆F. Find the number of ways of arranging the letters of the word TRIANGLE so that the relative positions of the vowels and consonants are not disturbed.

◆G Find the number of ways in which 4 letters can be put in 4 addressed envelopes so that no letter goes into the envelope meant for it.

◆H. Find the number of ways of arranging the letter of the words:
i) INTERMEDIATE ii) SINGING

◆I. Find the number of ways of arranging the letters of the word $a^4b^3c^5$ in its expanded form.

J. A man has 4 sons and there are 5 schools within his reach. In how many ways can he admit his sons in the schools so that no two of them will be in the same school?

K. Find the number of ways of arranging the letters of the word MONDAY so that no vowel occupies even place.

L. Find the number of 5-digit numbers that can be formed using the digits 1, 1, 2, 2, 3. How many of them are even?

M. Find the number of 7 digit numbers that can be formed using 2,2,2,3,3,4,4.

7.

◆A. If ${}^{12}C_{(s+1)} = {}^{12}C_{(2s-5)}$, find s.

◆B. If ${}^{n}C_{21} = {}^{n}C_{27}$, find ${}^{50}C_{n}$.

◆C. Find the number of ways of preparing a chain with 6 different coloured beads.

◆D. Find the number of ways of arranging the letters of the words.

(i) INDEPENDENCE (ii) MATHEMATICS

◆E. If $10.{}^{n}C_2 = 3.{}^{n+1}C_3$. Find n.

◆F. If ${}^{n}P_r = 5040$ and ${}^{n}C_r = 210$, find n and r.

◆G. If ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$, find r.

◆H. If ${}^{12}C_{r+1} = {}^{12}C_{3r-5}$, find r.

◆I. If ${}^{n}C_5 = {}^{n}C_6$, then find ${}^{13}C_n$.

◆J. Find the value of ${}^{10}C_5 + 2.{}^{10}C_4 + {}^{10}C_3$.

◆K. Simplify ${}^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4$.

◆L. Find the number of ways of selecting 3 vowels and 2 consonants from the letters of the word EQUATION.

8.

◆A. If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1+x)^{22}$ find the value of ${}^{13}C_r$.

◆B. Write down and simplify 6th term in $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$

◆C. Find the coefficient of

(i) x^{-6} in $\left(3x-\frac{4}{x}\right)^{10}$ (ii) x^{11} in $\left(2x^2+\frac{3}{x^3}\right)^{13}$ (iii) x^2 in $\left(7x^3-\frac{2}{x^2}\right)^9$ (iv) x^{-7} in $\left(\frac{2x^2}{3}-\frac{5}{4x^5}\right)^7$

◆D. Find the term independent of x in the expansion of

(i) $\left(\frac{\sqrt{x}}{3}-\frac{4}{x^2}\right)^{10}$ (ii) $\left(\frac{3}{\sqrt[3]{x}}+5\sqrt{x}\right)^{25}$ (iii) $\left(4x^3+\frac{7}{x^2}\right)^{14}$ (iv) $\left(\frac{2x^2}{5}+\frac{15}{4x}\right)^9$

◆E. Find the set of values of x for which the binomial expansions of the following are valid.

(i) $(2+3x)^{\frac{-2}{3}}$ (ii) $(5+x)^{\frac{3}{2}}$ (iii) $(7+3x)^{-5}$ (iv) $\left(4-\frac{x}{3}\right)^{\frac{-1}{2}}$

◆F. Find the number of terms in the expansion of $(2x+3y+z)^7$

◆G. Find the sum of last 20 coefficients in the expansion of $(1+x)^{39}$

◆H. If A and B are coefficient of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then find the value of $\frac{A}{B}$.

I. Find the middle term(s) in the expansion of

◆ (i) $\left(\frac{3x}{7}-2y\right)^{10}$ ◆ (ii) $\left(4x^2+5x^3\right)^{17}$ (iii) $\left(4a+\frac{3}{2}b\right)^{11}$ (iv) $\left(\frac{3}{a^3}+5a^4\right)^{20}$

J. Find the set E of the values of x for which the binomial expansions for the following are valid

◆(i) $(3-4x)^{\frac{3}{4}}$ ◆(ii) $(2+5x)^{\frac{-1}{2}}$ (iii) $(7-4x)^{-5}$ (iv) $(4+9x)^{\frac{-2}{3}}$ (v) $(a+bx)^r$

K. Find the largest binomial coefficient(s) in the expansion of (i) $(1+x)^{19}$ (ii) $(1+x)^{24}$

L. Find the 3rd term from the end in the expansion of $\left(x^{\frac{-2}{3}}-\frac{3}{x^2}\right)^8$. (Ans. $\frac{28\times 3^6}{x^{40/3}}$)

M. If the coefficients of $(2r+4)^{th}$ and $(r-2)^{nd}$ terms in the expansion of $(1+x)^{18}$ are equal, find r.

N. Find the numerically greatest term(s) in the expansion of

(i) $(4+3x)^{15}$ when $x=\frac{7}{2}$ (ii) $(3x+5y)^{12}$ when $x=\frac{1}{2}, y=\frac{4}{3}$

(iii) $(4a-6b)^{13}$ when $a=3, b=5$ (iv) $(3+7x)^n$ when $x=\frac{4}{5}, n=15$

O. If the coefficients of $(2r+4)^{th}$ term and $(3r+4)^{th}$ term in the expansion of $(1+x)^{21}$ are equal, find r.

9.

◆A. Find the mean deviation about the mean for the following data.

i) 38, 70, 48, 40, 42, 55, 63, 46, 54, 44 ii) 3, 6, 10, 4, 9, 10 iii) 6,7,10,12,13,4,12,16

◆B. Find the mean deviation about the median for the following frequency distribution.

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

◆C. An integer is picked from 1 to 20, both inclusive. Find the probability that it is a prime.

◆D. A fair coin is tossed 200 times. Find the probability of getting a head an odd number of times.

◆E. Out of 30 consecutive integers, two integers are drawn at random. Find the probability that their sum is odd.

◆F. Find the probability that a non-leap year contains (i) 53 Sundays and (ii) 52 Sundays only.

◆G. For any two events A and B, show that $P\left(A^c \cap B^c\right)=1+P(A\cap B)-P(A)-P(B)$.

H. Find the mean deviation about the median for the following data.

i) 13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17 ◆ii) 4, 6, 9, 3, 10, 13, 2

I. Find the mean deviation about the mean for the following distribution.

i)	x_i	10	11	12	13
	f_i	3	12	18	12

ii)	x_i	10	30	50	70	90
	f_i	4	24	28	16	8

J. Two dice are thrown, find the probability of getting the same number on both the faces.

K. A and B are among 20 persons who sit at random along a round table. Find the probability that there are any six persons between A and B.

L. If 4 fair coins are tossed simultaneously, then find the probability that 2 heads and 2 tails appear.

M. Two dice are rolled. What is the probability that none of the dice shows the number 2?

10.

◆A. 8 coins are tossed simultaneously. Find the probability of getting atleast 6 heads.

◆B. In a book of 450 pages, there are 400 typographical errors. Assuming that the number of errors per page follow the Poisson law, find the probability that a random sample of 5 pages will contain no typographical error.

◆C. A Poisson variable satisfies $P(X=1)=P(X=2)$. Find $P(X=5)$.

◆D. The probability of a bomb hitting a bridge is $\frac{1}{2}$ and three direct hits (not necessarily consecutive) are needed to destroy it. Find the minimum number of bombs required so that the probability of the bridge being destroyed is greater than 0.9.

◆E. If the mean and variance of a binomial variable X are 2.4 and 1.44 respectively, find $P(1<X\leq 4)$.

◆F. For a binomial distribution with mean 6 and variance 2, find the first two terms of the distribution.

◆G. Two dice are thrown, find the probability of getting the same number on both the faces.

◆H. A and B are among 20 persons who sit at random along a round table. Find the probability that there are any six persons between A and B.

◆I. If 4 fair coins are tossed simultaneously, then find the probability that 2 heads and 2 tails appear.

◆J. An integer is picked from 1 to 20, both inclusive. Find the probability that it is a prime.

◆K. A fair coin is tossed 200 times. Find the probability of getting a head an odd number of times.

◆L. Out of 30 consecutive integers, two integers are drawn at random. Find the probability that their sum is odd.

◆M. Find the probability of obtaining two tails and one head when 3 coins are tossed.

◆N. A page is opened at random from a book containing 200 pages. What is the probability that the number and page is a perfect square.

◆O. Find the probability that a non-leap year contains (i) 53 Sundays and (ii) 52 Sundays only.

◆P. Two dice are rolled. What is the probability that none of the dice shows the number 2?

Q. The mean and variance of a binomial distribution are 4 and 3 respectively. Fix the distribution and find $P(X \geq 1)$.

R. The probability that a person chosen at random is left handed (in hand writing) is 0.1. What is the probability that in a group of 10 people, there is one who is left handed.

S. Find the minimum number of times a fair coin must be tossed so that the probability of getting atleast one head is atleast 0.8.

T. If the difference between the mean and the variance of a binomial variate is $\frac{5}{9}$ then, find the probability for the event of 2 successes when the experiment is conducted 5 times.

U. One in 9 ships is likely to be wrecked, when they are set on sail, when 6 ships are on sail find the probability for (i) atleast one will arrive safely ii) exactly three will arrive safely.

V. It is given that 10% of the electric bulbs manufactured by a company are defective. In a sample of 20 bulbs, find the probability that more than 2 are defective.

W. On an average, rain falls on 12 days in every 30 days, find the probability that, rain will fall on just 3 days of a given week.

X. In a city 10 accidents take place in a span of 50 days. Assuming that the number of accidents follows the Poisson distribution, find the probability that there will be 3 or more accidents in a day.

Y. A bag contains 4 red,5 black and 6 blue balls. Find the probability that 2 balls are drawn from random from the bag or a red and black ball

SHORT ANSWER QUESTIONS

11.

◆A. If $x+iy=\frac{3}{2+\cos\theta+i\sin\theta}$ then, show that $x^2+y^2=4x-3$.

◆B. If $x+iy=\frac{1}{1+\cos\theta+i\sin\theta}$ then, show that $4x^2-1=0$.

◆C. If $z=3-5i$, then show that $z^3-10z^2+58z-136=0$.

◆D. If $(x-iy)^{\frac{1}{3}}=a-ib$, then show that $\frac{x}{a}+\frac{y}{b}=4\left(a^2-b^2\right)$.

◆E. If the point P denotes the complex number $z=x+iy$ in the Argand plane and if $\frac{z-i}{z-1}$ is a purely imaginary number, find the locus of P.

◆F. If $z=x+iy$ and if the point P in the Argand plane represents z, then describe geometrically the locus of z satisfying the equation $Arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$

G. Simplify $-2i(3+i)(2+4i)(1+i)$ and obtain the modulus of that complex number.

H. If the amplitude of $\left(\frac{z-2}{z-6i}\right)=\frac{\pi}{2}$, find its locus

12.

◆A. If x_1, x_2 are the roots of the quadratic equation $ax^2+bx+c=0$ and $c\neq 0$, find the value of $(ax_1+b)^{-2}+(ax_2+b)^{-2}$ in terms of a,b,c.

◆B. $\sqrt{\frac{x}{x-3}}+\sqrt{\frac{x-3}{x}}=\frac{5}{2}$, when $x\neq 0$ and $x\neq 3$

◆C. Show that none of the values of the function $\frac{x^2+34x-71}{x^2+2x-7}$ over $\mathbb{R}$ lies between 5 and 9.

◆D. Find the maximum value of the function $\frac{x^2+14x+9}{x^2+2x+3}$ over $\mathbb{R}$.

◆E. If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values for $x\in\mathbb{R}$, then find the bounds for p.

◆F. Prove that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if x is real.

G. Determine the range of the following expressions.

(i) $\frac{x^2+x+1}{x^2-x+1}$ ◆ (ii) $\frac{x+2}{2x^2+3x+6}$ ◆ (iii) $\frac{(x-1)(x+2)}{x+3}$ (iv) $\frac{2x^2-6x+5}{x^2-3x+2}$

H. Suppose that the quadratic equations $ax^2+bx+c=0$ and $bx^2+cx+a=0$ have common root. Then show that $a^3+b^3+c^3=3abc$.

I. If $c^2\neq ab$ and the roots of $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ are equal, then show that $a^3+b^3+c^3=3abc$ or $a=0$.

J. Let α and β be the roots of the quadratic equation $ax^2+bx+c=0$. If $c\neq 0$, then form the quadratic equation whose roots are $\frac{1-\alpha}{\alpha}$ and $\frac{1-\beta}{\beta}$.

13.

◆A. If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order, find the rank of the word PRISON.

◆B. Find the sum of all 4-digiti numbers that can be formed using the digits 1,3,5,7,9.

◆C. If the letters of the word EAMCET are permuted in all possible ways and if the words thus formed are arranged in the dictionary order, find the rank of the word EAMCET.

◆D. Find the sum of all 4 digited numbers that can be formed using the digits 0, 2, 4, 7, 8 without repetition.

◆E. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the ranks of the words

(i) REMAST (ii) MASTER

◆F. Find the number of ways of arranging 8 men and 4 women around a circular table. In how many of them i) all the women come together ii) no two women come together

◆G. If the letters of the word BRING are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the 59^{th} word.

H. Find the number of 4-letter words that can be formed using the letters of the word RAMANA.

I. If the letters of the word AJANTA are permuted in all possible ways and the words thus formed are arranged in dictionary order, find the ranks of the words (i) AJANTA (ii) JANATA

J. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 5, 6, 7. How many of them are divisible by (i) 2 (ii) 3 (iii) 4 (iv) 5 (v) 25

K. There are 9 objects and 9 boxes. Out of 9 objects, 5 cannot fit in three small boxes. How many arrangements can be made such that each object can be put in one box only.

L. Find the number of ways of seating 5 Indians, 4 Americans and 3 Russians at a round table so that i) all Indians sit together ii) no two Russians sit together

iii) persons of same nationality sit together

M. Find the number of ways of arranging 6 red roses and 3 yellow roses of different sizes into a garland. In how many of them

i) all the yellow roses are together ii) no two yellow roses are together

14.

◆A. Prove that $\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5....(4n-1)}{\{1.3.5.....(2n-1)\}^2}$.

◆B. Find the numbers of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.

◆C. Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans so that always the Indians will be in majority in the committee.

◆D. Find the number of 4-letters words that can be formed using the letters of the word RAMANA.

◆E. Find the number of ways of arranging the letters of the word ASSOCIATIONS. In how many of them i) all the three S's come together ii) the two A's do not come together

◆F. There are 8 railway stations along a railway line. In how many ways can a train be stopped at 3 of these stations such that no two of them are consecutive?

G. If a set A has 12 elements, find the number of subsets of A having

(i) 4 elements (ii) Atleast 3 elements (iii) Atmost 3 elements

H. Find the number of ways in which 12 things be (i) divided into 4 equal groups (ii) distributed to 4 persons equally

15.

Resolve the following into partial fractions:

◆A. $\frac{2x+3}{5(x+2)(2x+1)}$

◆B. $\frac{13x+43}{2x^2+17x+30}$

◆C. $\frac{3x-18}{x^3(x+3)}$

◆D. $\frac{x^2-2x+6}{(x-2)^3}$

◆E. $\frac{x^3+x^2+1}{\left(x^2+2\right)\left(x^2+3\right)}$

◆F. $\frac{2x^2+3x+4}{(x-1)\left(x^2+2\right)}$

◆G. $\frac{x^3}{(x-a)(x-b)(x-c)}$

◆H. $\frac{x^2+13x+15}{(2x+3)(x+3)^2}$

◆I. $\frac{x+4}{\left(x^2-4\right)(x+1)}$

◆J. $\frac{x^2}{(x-1)(x-2)}$

K. $\frac{5x+1}{(x+2)(x-1)}$

L. $\frac{x^2+5x+7}{(x-3)^3}$

M. $\frac{1}{(x-1)^2(x-2)}$

N. $\frac{x-1}{(x+1)(x-2)^2}$

O. $\frac{2x^2+2x+1}{x^3+x^2}$

P. $\frac{x^2-x+1}{(x+1)(x-1)^2}$

Q. $\frac{2x^2+1}{x^3-1}$

R. $\frac{x+3}{(1-x)^2(1+x)^2}$

S. $\frac{3x-1}{\left(1-x+x^2\right)(x+2)}$

T. $\frac{x^4}{(x-1)(x-2)}$

U. $\frac{x^3}{(x-1)(x+2)}$

V. $\frac{x^3}{(2x-1)(x-1)^2}$

16.

◆A. Two squares are chosen at random on a chess board. Show that the probability that they have a side in common is $\frac{1}{18}$.

◆B. A and B are seeking admission into IIT. If the probability for A to be selected is 0.5 and that both to be selected is 0.3, then it is possible that, the probability of B to be selected is 0.9?

◆C. The probability for a contractor to get a road contract is $\frac{2}{3}$ and to get a building contract is $\frac{5}{9}$. The probability to get atleast one contract is $\frac{4}{5}$. Find the probability that he gets both the contracts.

◆D. Two persons A and B are rolling a die on the condition that the person who gets 3 will win the game. If A starts the game, then find the probabilities of A and B respectively to win the game.

◆E. The probabilities of three events A, B, C are such that $P(A)=0.3$, $P(B)=0.4$, $P(C)=0.8$, $P(A\cap B)=0.08$, $P(A\cap C)=0.28$, $P(A\cap B\cap C)=0.09$ and $P(A\cup B\cup C)\geq 0.75$. Show that $P(B\cap C)$ lies in the interval [0.23, 0.48].

◆F. The probabilities of three mutually exclusive events are respectively given as $\frac{1+3p}{3},\frac{1-p}{4},\frac{1-2p}{2}$. Prove that $\frac{1}{3}\leq p\leq\frac{1}{2}$.

◆G. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, find the probability that

i) you both enter the same section   ii) you both enter the different sections.

◆H. If one ticket is randomly selected from tickets numbered 1 to 30, then find the probability that the number on the ticket is (i) a multiple of 5 or 7 (ii) a multiple of 3 or 5.

◆I. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that the sum of the two numbers is (i) an even number (ii) an odd number.

J. Find the probability of throwing a total score of 7 with 2 dice.

K. In a box containing 15 bulbs, 5 are detective. If 5 bulbs are selected at random from the box, find the probability of the event, that

i) none of them is defective ii) only one of them is defective iii) atleast one of them is defective

L. In a committee of 25 members, each member is proficient either in mathematics or in statistics or in both. If 19 of these are proficient in mathematics, 16 in statistics, find the probability that a person selected from the committee is proficient in both.

M. A bag contains 12 two rupee coins, 7 one rupee coins and 4 half a rupee coins. If three coins are selected at random, then find the probability that

i) the sum of three coins is maximum ii) the sum of three coins is minimum

iii) each coin is different value

N. A, B, C are three horses in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are the probabilities of A, B and C to win the race?

O. On a festival day, a man plans to visit 4 holy temples A,B,C,D in a random order. Find the probability that he visits (i) A before B (ii) A before B and B before C.

P. For any two events A and B, show that $P\left(A^c \cap B^c\right) = 1 + P(A \cap B) - P(A) - P(B)$.

Q. A pair of dice is rolled. What is the probability that they sum to 7 given that neither die shows a 2?

R. An urn contains 12 red balls and 12 green balls. Suppose two balls are drawn one after another without replacement. Find the probability that the second ball drawn is green, given that the first ball drawn is red.

17.

◆A. A pair of dice is thrown. Find the probability that either of the dice shows 2 when their sum is 6.

◆B. A speaks truth in 75% of the cases and B in 80% cases. What is the probability that their statements about an incident do not match.

◆C. A and B toss a coin 50 times each simultaneously. Find the probability that both of them will not get tails at the same toss.

◆D. If A, B, C are three independent events of an experiment such that, $P\left(A \cap B^c \cap C^c\right) = \frac{1}{4}$, $P\left(A^c \cap B^c \cap C^c\right) = \frac{1}{8}$, $P\left(A^c \cap B^c \cap C^c\right) = \frac{1}{4}$, then find $P(A)$, $P(B)$ and $P(C)$.

◆E. The probability that Australia wins a match against India in a cricket game is given to be $\frac{1}{3}$. If India and Australia play 3 matches, what is the probability that (i) Australia will loose all the three matches? (ii) Australia will win atleast one match?

◆F. Suppose that an urn B_1 contains 2 white and 3 black balls and another urn B_2 contains 3 white and 4 black balls. One urn is selected at random and a ball is drawn from it. If the ball drawn is found black, let us find the probability that the urn chosen was B_1.

◆G. A problem in Calculus is given to two students A and B whose chances of solving it are $\frac{1}{3}$ and $\frac{1}{4}$ respectively. Find the probability of the problem being solved if both of them try independently.

◆H. There are 3 black and 4 white balls in one bag, 4 black and 3 white balls in the second bag. A die is rolled and the first bag is selected if the die shows up 1 or 3, and the second bag for the rest. Find the probability of drawing a black ball, from the bag thus selected.

◆I. Suppose A and B are independent events with $P(A)=0.6, P(B)=0.7$ then compute

i) $P(A\cap B)$ ii) $P(A\cup B)$ iii) $P\left(\frac{B}{A}\right)$ iv) $P\left(A^c \cap B^c\right)$

J. Suppose there are 12 boys and 4 girls in a class. If we choose three children one after another in succession at random, find the probability that all the three are boys.

K. An urn contains w white balls and b black balls. Two players Q and R alternatively draw a ball with replacement from the urn. The player that draws a white ball first wins the game. If Q begins the game, find the probability of his winning the game.

LONG ANSWER QUESTIONS

18.

◆A. If n is an integer then show that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$.

◆B. If $\cos\theta+\cos\beta+\cos\gamma=0$ $\cos\theta+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, prove that

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=\frac{3}{2}=\sin^2\alpha+\sin^2\beta+\sin^2\gamma$.

◆C. Find all the roots of the equation $x^{11}-x^7+x^4-1=0$.

◆D. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$ then show that

(i) $\cos 3\alpha+\cos 3\beta+\cos 3\gamma=3\cos(\alpha+\beta+\gamma)$ (ii) $\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$

(iii) $\cos(\alpha+\beta)+\cos(\beta+\gamma)+\cos(\gamma+\alpha)=0$

◆E. Solve $x^9-x^5+x^4-1=0$

◆F. Prove the sum of 99th powers of the roots of the equation $x^7-1=0$ is zero and hence deduce the roots of $x^6+x^5+x^4+x^3+x^2+x+1=0$.

◆G. If n is a positive integer, show that $(P+iQ)^{\frac{1}{n}}+(P-iQ)^{\frac{1}{n}}=2\left(P^2+Q^2\right)^{\frac{1}{2n}}.\cos\left[\frac{1}{n}\tan^{-1}\frac{Q}{P}\right]$.

◆H. Show that one value of $\left[\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^{\frac{8}{3}}$ is -1.

◆I. If n is a positive integer, show that $(1+i)^n + 1(1-i)^n = 2^{\frac{n+2}{2}} \cos\left(\frac{n\pi}{4}\right)$

◆J. If n is an integer then show that $(1+i)^{2n} + (1-i)^{2n} = 2^{n+1} \cos\frac{n\pi}{2}$.

◆K. If α, β are the roots of the equation $x^2 - 2x + 4 = 0$ then for any $n \in N$ show that $\alpha^n + \beta^n = 2^{n+1} \cos\left(\frac{n\pi}{3}\right)$.

19.

◆A. Find the roots of $x^4 - 16x^3 + 86x^2 - 176x + 105 = 0$.

◆B. Solve $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots of this equation are in arithmetic progression.

◆C. Solve the equation $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

◆D. Solve the equation $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$.

◆E. Solve the equation $6x^6 - 25x^5 + 31x^4 - 31x^2 + 25x - 6 = 0$.

◆F. Solve $x^4 - 4x^2 + 8x + 35 = 0$, given that $2 + i\sqrt{3}$ is a root.

◆G. Find the polynomial equation whose roots are the translates of those of the equation $x^5 - 4x^4 + 3x^2 - 4x + 6 = 0$ by -3.

H. Solve the following equations, given that the roots of each are in A.P.

◆ (i) $8x^3 - 36x^2 - 18x + 81 = 0$ (ii) $x^3 - 3x^2 - 6x + 8 = 0$

I. Solve the following equations, given that the roots of each are in G.P

◆ (i) $54x^3 - 39x^2 - 26x + 16 = 0$ (ii) $3x^3 - 26x^2 + 52x - 24 = 0$

J. Solve the following equations.

◆ (i) $2x^5 + x^4 - 12x^3 - 12x^2 + x + 2 = 0$ (ii) $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

K. Find the polynomial equation whose roots are the translates of the roots of the equation $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2.

L. Solve the following equations, given that the roots of each are in H.P

(i) $6x^3 - 11x^2 + 6x - 1 = 0$ (ii) $15x^3 - 23x^2 + 9x - 1 = 0$

20.

◆A. For $r = 0, 1, 2, \ldots, n$, prove that $C_0.C_r + C_1.C_{r+1} + C_2.C_{r+2} + \ldots\ldots + C_{n-r}.C_n = {}^{2n}C_{(n+r)}$ and hence deduce that (i) $C_0^2 + C_1^2 + \ldots\ldots + C_n^2 = {}^{2n}C_n$ (ii) $C_0.C_1 + C_1.C_2 + C_2.C_3 + \ldots\ldots + C_{n-1}.C_n = {}^{2n}C_{n+1}$

◆B. Suppose that n is a natural number and, I, F are respectively that integral part and fractional part of $\left(7 + 4\sqrt{3}\right)^n$. Then show that (i) I is an odd integer (ii) $(I + F)(1 - F) = 1$.

◆C. If n is a positive integer and x is any nonzero real number, then prove that

$$C_0 + C_1.\frac{x}{2} + C_2.\frac{x^2}{3} + C_3.\frac{x^3}{4} + \ldots\ldots + C_n.\frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

◆D. If the coefficients of x^9, x^{10}, x^{11} in the expansion of $(1+x)^n$ are in A.P. then prove that $n^2 - 41n + 398 = 0$.

◆E. If the 2nd, 3rd and 4th terms in the expansion of $(a+x)^n$ are respectively 240, 720, 1080, find a, x, n.

◆F. If P and Q are the sum of even terms and odd terms respectively in the expansion of $(x+a)^n$ then prove that (i) $P^2 - Q^2 = \left(x^2 - a^2\right)^n$ (ii) $4PQ = (x+a)^{2n} - (x-a)^{2n}$

◆G. If the coefficients of 4 consecutive terms in the expansion $(1+x)^n$ are a_1, a_2, a_3, a_4 respectively, then show that $\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$.

◆H. If $\left(1 + 3x - 2x^2\right)^{10} = a_0 + a_1 x + a_2 x^2 + ... + a_{20} x^{20}$, then prove that

(i) $a_0 + a_1 + a_2 + ...a_{20} = 2^{10}$ (ii) $a_0 - a_1 + a_2 - a_3 + ... + a_{20} = 4^{10}$

◆I. If I, n are positive integers, $0 < f < 1$ and if $\left(7 + 4\sqrt{3}\right)^n = I + f$, then show that

(i) I is an odd integer and (ii) $(I + f)(1 - f) = 1$

◆J. If $\left(1 + x + x^2\right) = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$, then prove that

(i) $a_1 + a_3 + a_5 + ...a_{2n-1} = \frac{3^n - 1}{2}$ (ii) $a_0 + a_3 + a_6 + a_9 + ... = 3^{n-1}$

(iii) $a_0 + a_1 + a_2 + ...a_{2n} = 3^n$ (iv) $a_0 + a_2 + a_4 + ...a_{2n} = \frac{3^n + 1}{2}$

K. If R, n are positive integers, n is odd, $0 < F < 1$ and if $\left(5\sqrt{5} + 11\right)^n = R + F$ then prove that

(i) R is an even integer and (ii) $(R + F).F = 4^n$

L. Prove that $6^{2n} - 35n - 1$ is divisible by 1225 for all natural numbers n.

M. If 36, 84, 126 are three successive binomial coefficients in the expansion of $(1+x)^n$, then find n.

N. If the coefficients of $r^{th}, (r+1)^{th}$, and $(r+2)^{nd}$ terms in the expansion of $(1+x)^n$ are in A.P., then show that that $n^2 - (4r+1)n + 4r^2 - 2 = 0$.

O. Find the sum of the coefficients of x^{32} and x^{-18} in the expansion of $\left(2x^3 - \frac{3}{x^2}\right)^{14}$.

P. Prove that $\left({}^{2n}C_0\right)^2 - \left({}^{2n}C_1\right)^2 + \left({}^{2n}C_2\right)^2 - \left({}^{2n}C_3\right)^2 + ... + \left({}^{2n}C_{2n}\right)^2 = (-1)^n . {}^{2n}C_n$.

Q. Prove that $(C_0 + C_1)(C_1 + C_2)(C_2 + C_3)....(C_{n-1} + C_n) = \frac{(n+1)^n}{n!}.C_0.C_1.C_2.........C_n$.

R. If n is a positive integer, prove that $\sum_{r=1}^{n} r^3 \left(\frac{{}^nC_r}{{}^nC_{r-1}}\right)^2 = \frac{n(n+1)^2 (n+2)}{12}$

S. If the coefficient of x^{10} in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-10} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$, find the relation between a and b where a and b and real numbers.

21.

◆A. Find the sum of the series $\frac{3.5}{5.10}+\frac{3.5.7}{5.10.15}+\frac{3.5.7.9}{5.10.15.20}+.....\infty$.

◆B. If $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+.....\infty$, then find $3x^2+6x$.

◆C. If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+.....,$ then prove that $9x^2+24x=11$.

◆D. If $x=\frac{5}{(2!)3}+\frac{5.7}{(3!).3^2}+\frac{5.7.9}{(4!).3^3}+....,$ then find the value of x^2+4x.

◆E. Find the sum of infinite terms of the series $\frac{7}{5}\left(1+\frac{1}{10^2}+\frac{1.3}{1.2}.\frac{1}{10^4}+\frac{1.3.5}{1.2.3}.\frac{1}{10^6}+...\right)$

F. Find the sum of the infinite series

◆ (i) $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+....$

◆ (ii) $\frac{3}{4}+\frac{3.5}{4.8}+\frac{3.5.7}{4.8.12}+....$

(iii) $1-\frac{4}{5}+\frac{4.7}{5.10}-\frac{4.7.10}{5.10.15}+....$

(iv) $\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}-....$

G. Find the coefficient of x^6 in the expansion of $(1-3x)^{\frac{-2}{5}}$.

H. Find the sum of the infinite series $1+\frac{2}{3}.\frac{1}{2}+\frac{2.5}{3.6}\left(\frac{1}{2}\right)^2+\frac{2.5.8}{3.6.9}\left(\frac{1}{2}\right)^3+....\infty$.

I. If $t=\frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+....\infty$, then prove that $9t=16$.

J. Show that for any non zero rational number x.

$$1+\frac{x}{2}+\frac{x(x-1)}{2.4}+\frac{x(x-1)(x-2)}{2.4.6}+.... = 1+\frac{x}{3}+\frac{x(x+1)}{3.6}+\frac{x(x+1)(x+2)}{3.6.9}+....$$

22.

◆A. Find the mean deviation about the mean for the following data:

Marks obtained	0-10	10-20	20-30	30-40	40-50
No. of students	5	8	15	16	6

◆B. Find the mean deviation from the median for the following data

Age (years)	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No. of workers (f_i)	120	125	175	160	150	140	100	30

◆C. Calculate the variance and standard deviation of the following continuous frequency distribution

Class interval	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

◆D. The following table gives the daily wages of workers in a factory. Compute the standard deviation and the coefficient of variation of the wages of the workers.

Wages (Rs.)	125-175	175-225	225-275	275-325	325-375	375-425	425-475	475-525	525-575
Number of workers	2	22	19	14	3	4	6	1	1

◆E. Find the mean deviation about the median for the following continuous distribution:

i)

Marks obtained	0-10	10-20	20-30	30-40	40-50	50-60
No. of boys	6	8	14	16	4	2

ii)

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	5	8	7	12	28	20	10	10

◆F. Find the mean deviation about the mean for the following continuous distribution

Height (in cms)	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

◆G. Find the mean and variance using the step deviation method, of the following tabular data, giving the age distribution of 542 members.

Age in years (x_i)	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Number of members (f_i)	3	61	132	153	140	51	2

H. Find the mean deviation from the mean for the following continuous frequency distribution

Sales (In Rs. thousand)	40-50	50-60	60-70	70-80	80-90	90-100
Number of companies	5	15	25	30	20	5

I. Find the variance and standard deviation for the following discrete frequency distribution:

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

J. Lives of two models of refrigerators A and B, obtained in a survey, are given below:

Life (in years)	Model A	Model B
0-2	5	2
2-4	16	7
4-6	13	12
6-8	7	19
8-10	5	9

K. Find the mean deviation from the mean of the following data, using the step deviation method

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	6	5	8	15	7	6	3

L. The scores of two cricketers A and B in 10 innings are given below. Find who is a better run getter and who is a more consistent player.

Scores of A : x_i	40	25	19	80	38	8	67	121	66	76
Scores of B : y_i	28	70	31	0	14	111	66	31	25	4

M. Find the variance and standard deviation of the following frequency distribution

x_i	6	10	14	18	24	28	30
f_i	2	4	7	12	8	4	3

N. From the prices of shares X and Y given below, for 10 days of trading, find out which share is more stable?

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

23.

◆A. State and prove Baye's theorem.

◆B. Three boxes numbered I, II, III contain the balls as follows:

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II

◆C. In a shooting test the probability of A, B, C hitting the targets are $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ respectively. If all of them fire at the same target, find the probability that (i) only one of them hits the target, (ii) atleast one of them hits the target.

◆D. State and prove addition theorem of probability.

◆E. State and prove multiplication theorem of probability.

◆F. Three boxes B_1, B_2 and B_3 contains the balls with different colours as shown below:

	White	Black	Red
B_1	2	1	2
B_2	3	2	4
B_3	4	3	2

A die is thrown. B_1 is chosen if either 1 or 2 turns up. B_2 is chosen if 3 or 4 turns up and B_3 is chosen if 5 or 6 turns up. Having chose a box in this way, a ball is chosen at random from this box. If the ball drawn is found to be red, find the probability that it is drawn from box B_2.

◆G. Three Urns have the following composition of balls.

Urn I : 1 white, 2 black

Urn II : 2 white, 1 black

Urn III : 2 white, 2 black

One of the urns is selected at random and a ball is drawn. It turns out to be white. Find the probability that it came from urn III.

H. A person is known to speak truth 2 out of 3 times. He throws a die and reports that it is 1. Find the probability that is actually 1.

24.

◆A. The probability distribution of a random variable X is given below:

$X = x_i$	1	2	3	4	5
$P(X = x_i)$	k	$2k$	$3k$	$4k$	$5k$

Find the value of k and the mean and variance of X.

◆B. Let X be a random variable such that $P(X=-2)=P(X=-1)=P(X=2)=P(X=1)=\frac{1}{6}$ and $P(X=0)=\frac{1}{3}$. Find the mean and variance of X.

◆C.

$X = x$	-2	-1	0	1	2	3
$P(X = x)$	0.1	k	0.2	$2k$	0.3	k

is the probability distribution of random variable X. Find the value of k and variance of X.

◆D. Two dice are rolled at random. Find the probability distribution of the sum of the numbers on them. Find the mean of the random variable.

◆E. A random variable X has the following probability distribution.

$X=x$	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) the mean and (iii) $P(0<X<5)$

◆F. The range of a random variable X is $\{0,1,2\}$. Given that $P(X=0)=3c^3$, $P(X=1)=4c-10c^2$, $P(X=2)=5c-1$. i) Find the value of c ii) $P(X<1)$, $P(1<X\leq 2)$ and $P(0<X\leq 3)$.

◆G. The range of a random variable X is $\{1,2,3,.....\}$ and $P(X=k)=\frac{c^k}{k!}$; $(k=1,2,3,....)$. Find the value of c and $P(0<X<3)$.

H. If x is a random variable with probability distribution $P(X=k)=\frac{(k+1)c}{2^k}$, $k=0,1,2,...$ then find c.

I.

$X=x$	-3	-2	-1	0	1	2	3
$P(X=x)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

is the probability distribution of a random variable X. Find the variance of X.

wish you all the best