KUKATPALLY CENTRE

IMPORTANT QUESTIONS

FOR

INTERMEDIATE PUBLIC EXAMINATIONS

IN

MATHS-IB

2017-18

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Total No. of Questions - 24 **Reg. No.**

Total No. of Printed Pages - 2

Part - III
MATHEMATICS, Paper-I (B)
(English Version)

Time : 3 Hours] **[Max. Marks : 75**

SECTION - A $\quad$ 10 × 2 = 20 M

I. Very Short Answer Type questions:

1. Find the value of 'y', if the line joining the points $(3, y)$ and $(2, 7)$ is parallel to the line joining the points $(-1, 4)$, $(0, 6)$.

2. Find the value of $\frac{n^2}{\sqrt{3}\left(l^2+m^2\right)}$, if the straight lines $x+p=0$, $y+2=0$ and $3x+2y+5=0$ are concurrent.

3. Find the fourth vertex of the parallelogram whose consecutive vertices are $(2,4,-1),(3,6,-1)$ and $(4,5,1)$.

4. Find the angle between the planes $x+2y+2z-5=0$ and $3x+3y+2z-8=0$.

5. Compute $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$.

6. Compute $\lim\limits_{x\to\infty} \frac{8|x|+3x}{3|x|-2x}$.

7. If $f(x)=7^{x^3+3x}\,(x>0)$, then find $f'(x)$.

8. If $x=\tan\left(e^{-y}\right)$, then show that $\frac{dy}{dx}=\frac{-e^{y}}{1+x^2}$

9. Find dy and Δy of $y=x^2+x$ at $x=10$ when $\Delta x=0.1$.

10. Verify Rolle's theorem for the function $f:[-3,8]\to\mathbb{R}$ be defined by $f(x)=x^2-5x+6$.

SECTION – B $\quad$ 5 × 4 = 20 M

II. Short Answer Type questions:

(i) Attempt any **five** questions

(ii) Each question carries **four** marks

11. $A(5,3)$ and $B(3,-2)$ are two fixed points. Find the equation of locus of P, so that the area of ΔPAB is 9 sq. units.

12. When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of $3x^2+10xy+3y^2=9$.

13. $x-3y-5=0$ is the perpendicular bisector of the line segment joining the points A, B. If $A=(-1,-3)$, find the coordinates of 'B'.

14. Show that $f(x)=\begin{cases}\frac{\cos ax-\cos bx}{x^2} & \text{if} \quad x\neq 0\\ \frac{1}{2}\left(b^2-a^2\right) & \text{if} \quad x=0\end{cases}$

    where a and b are real constants, is continuous at $x=0$.

15. If $ay^4 = (x+b)^5$ then $5yy'' = (y')^2$.

16. Find the lengths of subtangent, subnormal at a point 't' on the curve $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$.

17. The volume of a cube is increasing at rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of the edge is 10 centimetres?

SECTION – C $\quad$ 5 × 7 = 35 M

III. Long Answer Type questions:
(i) Attempt any **five** questions
(ii) Each question carries **seven** marks

18. Find the orthocentre of the triangle whose vertices are $(5,-2)$, $(-1,2)$ and $(1,4)$.

19. Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and the line $lx + my + n = 0$ is $\left|\dfrac{n^2\sqrt{h^2 - ab}}{am^2 - 2h/m + bl^2}\right|$.

20. The condition for the line joining the origin to the point of intersection of the circle $x^2 + y^2 = a^2$ and the line $lx + my = 1$ to coincide.

21. Find the direction consines of two lines which are connected the relation $l + m + n = 0$ and $mn - 2nl - 2lm = 0$.

22. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ then prove that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$.

23. At a point (x_1, y_1) on the curve $x^3 + y^3 = 3axy$, show that the tangent is $(x_1^2 - ay_1)x + (y_1^2 - ax_1)y = ax_1y_1$.

24. A window is in the shape of rectangle surmounted by a semicircle. If the perimeter of the window is 20 ft. find the maximum area.

BLUE PRINT (MATHS-IB)

S.No.	Name of the chapter	Weightage Marks
	CO-ORDINATE GEOMETRY	
1.	Locus	4 (4)
2.	Transformation of axes	4 (4)
3.	Straight line	15 (7 + 4 + 2+ 2)
4.	Pair of straight lines	14 (7 + 7)
	3D GEOMETRY	
5.	3D-coordinates	2 (2)
6.	Direction Cosines & Direction Rations	7 (7)
7.	The plane	2 (2)
	CALCULUS	
8.	Limits & Continuity	8 (4 + 2 + 2)
9.	Differentiation	15 (7 + 4 + 2 + 2)
10.	Errors – Approximations	2 (2)
11.	Tangent & Normal	11 (7 + 4)
12.	Rate measure	4 (4)
13.	Rolle's & Lagrange's Theorems	2 (2)
14.	Maxima & Minima	7 (7)

VERY SHORT ANSWER QUESTIONS

1.

◆A. Find the equation of the straight line passing through $(-2,4)$ and making non zero intercepts whose sum is zero.

◆B. Find the equation of the straight line passing through $(3,-4)$ and making X and Y-intercepts which are in the ratio 2 : 3.

◆C. If the area of the triangle formed by the straight lines $x=0$, $y=0$ and $3x+4y=a$ $(a>0)$ is 6. Find the value of a.

◆D. Transform the equation $x+y+1=0$ into normal form.

◆E. Find the ratio in which the line $2x+3y-20=0$ divides the join of the points (2, 3) and (2, 10).

◆F. Prove that the points (-5, 1), (5, 5), (10, 7) are collinear and find the equation of the line containing these points.

◆G. If the portion of a straight line intercepted between the axes of coordinates is bisected at $(2p,2q)$. Find the equation of the straight line.

H. Find the equations of the straight line passing through the following points :

a) (2, 5), (2, 8) b) (3, -3), (7, -3) c) (1, -2), (-2, 3) d) $\left(at_1^2, 2at_1\right)\left(at_2^2, 2at_2\right)$

I. Find the equations of the straight lines passing through the point (4,–3) and (i) parallel (ii) perpendicular to the line passing through the points (1, 1) and (2, 3).

J. Find the equation of straight line passing through origin and making equal angles with coordinate axes.

K. Find the equation of the straight line making an angle of $Tan^{-1}\left(\frac{2}{3}\right)$ with the positive x-axis and has y-intercept 3.

L. A straight line passing through $A(-2,1)$ makes an angle of 30° with $\overleftrightarrow{OX}$ in the positive direction. Find the points on the straight line whose distance from A is 4 units.

M. Find the points on the line $3x-4y-1=0$ which are at a distance of 5 units from the point (3, 2).

N. Find the equation of line which makes an angle of 150° with positive x-axis and passing through (-2, -1).

O. State whether (3, 2) and (-4, -3) are on the same side or on opposite side of the straight line $2x-3y+4=0$

2.

◆A. Find the area of the triangle formed by the coordinate axes and the line $3x-4y+12=0$

◆B. Find the set of values of 'a' if the points (1, 2) and (3, 4) lie on the same side of the straight line $3x-5y+a=0$

◆C. Find the distance between the straight lines $5x-3y-4=0$ and $10x-6y-9=0$.

◆D. Find the value of 'k' if the straight lines $y-3kx+4=0$ & $(2k-1)x-(8k-1)y-6=0$ are perpendicular.

◆E. Find the orthocenter of the triangle whose sides are given by $x+y+10=0$, $x-y-2=0$ and $2x+y-7=0$.

◆F. If a, b, c are in A.P, show that $ax+by+c=0$ represents a family of concurrent lines and find the point of concurrency.

◆G. Find the equation of the line perpendicular to the line $3x+4y+6=0$ and making an intercept –4 on the x-axis.

◆H. Find the point of concurrency of the lines represented by $(2+5k)x-3(1+2k)y+(2-k)=0$

◆I. Find the equation of the straight line passing through the point of intersection of the lines $x+y+1=0$ and $2x-y+5=0$ and containing the point (5, -2).

J. If $2x-3y-5=0$ is the perpendicular bisector of the line segment joining (3, –4) and (α,β). Find $\alpha+\beta$.

K. Find the length of the perpendicular drawn from (3, 4) to the line $3x-4y+10=0$.

L. Find the circumcentre of the triangle formed by the lines $x=1$, $y=1$ and $x+y=1$.

M. Find the value of k, if the angle between the straight lines $4x-y+7=0$ and $kx-5y-9=0$ is 45°.

N. If (-2, 6) is the image of the point (4, 2) w.r.t the line L, then find the equation of L.

O. Find the angle which the straight line $y=\sqrt{3x}-4$ makes with y-axis.

3.

◆A. Find x if the distance between $(5,-1,7)$ and $(x,5,1)$ is 9 units.

◆B. Show that the points $(1,2,3),(7,0,1)$ and $(-2,3,4)$ are collinear.

◆C. Show that the points $(2,3,5)$, $(-1,5,-1)$ and $(4,-3,2)$ form a right angled isosceles triangle.

◆D. Show that the points $(1,2,3),(2,3,1)$ and $(3,1,2)$ form an equilateral triangle.

◆E. P is a variable point which moves such that *3PA = 2PB*. If A = $(-2,2,3)$ and $B=(13,-3,13)$, prove that P satisfies the equation $x^2+y^2+z^2+28x-12y+10z-247=0$

◆F. Show that *ABCD* is a square where *A, B, C, D* are the points $(0,4,1),(2,3,-1),(4,5,0)$ and $(2,6,2)$ respectively.

4.

◆A. Find the equation of the plane if the foot of the perpendicular from origin to the plane is $(2,3,-5)$.

◆B. Reduce the equation $x+2y-3z-6=0$ of the plane to the normal form.

◆C. Find the angle between the planes $x+2y+2z-5=0$ and $3x+3y+2z-8=0$.

◆D. Find the equation to the plane parallel to the ZX -plane and passing through $(0,4,4)$.

◆E. Find the equation of the plane passing through the point $(-2,1,3)$ and having $(3,-5,4)$ as d.r.'s of its normal.

◆F. Find the equation of the plane passing through the point $(1,1,1)$ and parallel to the plane $x+2y+3z-7=0$.

G. Find the equation of the plane through $(4,4,0)$ and perpendicular to the planes $2x+y+2z+3=0$ and $3x+3y+2z-8=0$.

H. Find the equation of the plane passing through $(2,0,1)$ and $(3,-3,4)$ and perpendicular to $x-2y+z=6$.

I. Find the equation of the plane through the points $(2,2,-1),(3,4,2),(7,0,6)$.

J. A plane meets the coordinate axes in A,B,C. If centroid of the ΔABC is (a,b,c). Show that the equation to the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3$.

5.

◆A. Find $\underset{x\to 0}{Lt}\frac{\sqrt{1+x}-1}{x}$

◆B. Compute $\underset{x\to 0}{Lt}\frac{e^x-1}{\sqrt{1+x}-1}$

◆C. Compute $\underset{x\to \pi/2}{Lt}\frac{\cos x}{x-\pi/2}$

◆D. Compute $\underset{x\to 0}{Lt}\frac{\sin(a+bx)-\sin(a-bx)}{x}$

◆E. Compute $\underset{x\to 2+}{Lt}[x]+x$ and $\underset{x\to 2-}{Lt}[x]+x$

◆F. Compute $\underset{x\to 0}{Lt}\frac{a^x-1}{b^x-1}$,$(a>0,b>0,b\neq 1)$

◆G. Compute $\underset{x\to 0}{Lt}\frac{\sin ax}{\sin bx}$,$b\neq 0$, $a\neq b$

◆H. Evaluate $\underset{x\to 0}{Lt}x^2\cos\frac{2}{x}$

◆I. Compute $\underset{x\to 0}{Lt}\frac{e^x-\sin x-1}{x}$

J. Compute $\underset{x\to 3}{Lt}\frac{x^2-8x+15}{x^2-9}$

K. If $f(x)=\begin{cases}x^2 & , x\leq 1\\ 2x-1, & x>1\end{cases}$ then find $\underset{x\to 1+}{Lt}f(x)$ and $\underset{x\to 1-}{Lt}f(x)$

L. Compute $\underset{x\to 0}{Lt}\ \frac{e^{3x}-1}{x}$

M. Find $\underset{x\to 0}{Lt}\ f(x)$ where $f(x)=\begin{cases} x-1 & if\ x<0 \\ 0 & if\ x=0 \\ x+1 & if\ x>0 \end{cases}$

N. Evaluate $\underset{x\to 1}{Lt}\ \frac{\log_e x}{x-1}$

O. Compute $\underset{x\to 1}{Lt}\ \frac{\sin(x-1)}{x^2-1}$

P. Compute $\underset{x\to 0}{Lt}\ \frac{\tan(x-a)}{x^2-a^2}$ $(a\neq 0)$

Q. Compute $\underset{x\to 3}{Lt}\ \frac{e^x-e^3}{x-3}$

6.

◆A. Compute $\underset{x\to 0}{Lt}\ \frac{x(e^x-1)}{1-\cos x}$

◆B. Compute $\underset{x\to\infty}{Lt}\ \frac{8|x|+3x}{3|x|-2x}$

◆C. Compute $\underset{x\to\infty}{Lt}\ \frac{2+\cos^2 x}{x+2007}$

◆D. Is f defined by $f(x)=\begin{cases} \frac{\sin 2x}{x}, & if\ x\neq 0 \\ 1, & if\ x=0 \end{cases}$ continuous at $x=0$

◆E. Show that $f(x)=\begin{cases} \frac{\cos ax-\cos bx}{x^2} & if\ x\neq 0 \\ \frac{1}{2}(b^2-a^2) & if\ x=0 \end{cases}$ where a and b are real constants, is continuous at 0

◆F. Compute $\underset{x\to 0}{Lt}\ \frac{1-\cos mx}{1-\cos nx}$ $n\neq 0$

◆G. Compute $\underset{x\to -\infty}{Lt}\ \frac{2x+3}{\sqrt{x^2-1}}$

◆H. Compute $\underset{x\to\infty}{Lt}\ \frac{2+\sin x}{x^2+3}$

I. Compute $\underset{x\to\infty}{Lt}\ \frac{\sqrt{x^2+6}}{2x^2-1}$

J. Compute $\underset{x\to -\infty}{Lt}\ \frac{5x^3+4}{\sqrt{2x^4+1}}$

K. Compute $\underset{x\to\infty}{Lt}\ \frac{\cos x+\sin^2 x}{x+1}$

7.

◆A. If $f(x) = x\, e^x \sin x$, then find $f'(x)$

◆B. If $f(x) = 1 + x + x^2 + ... + x^{100}$ then find $f'(1)$

◆C. If $f(x) = \log(\sec x + \tan x)$ find $f'(x)$

◆D. If $f(x) = 7^{x^2+3x}\ (x > 0)$ then find $f'(x)$

◆E. If $f(x) = \sin(\log x)\ ,(x > 0)$ find $f'(x)$

◆F. If $f(x) = \left(x^3 + 6x^2 + 12x - 13\right)^{100}$ find $f'(x)$

◆G. If $f(x) = \log_7(\log x)\ , x > 0$ find $\frac{dy}{dx}$

◆H. If $f(x) = 2x^2 + 3x - 5$ then prove that $f'(0) + 3f'(-1) = 0$

8.

◆A. If $y = \frac{2x+3}{4x+5}$ then find y''

◆B. If $y = a\, e^{nx} + b\, e^{-nx}$ then prove that $y'' = n^2 y$

◆C. If $y = \log\left(\sin(\log x)\right)$, find $\frac{dy}{dx}$

◆D. Find the derivative of $\log\left(\frac{x^2+x+2}{x^2-x+2}\right)$

◆E. Find the second order derivative of $f(x) = \log\left(4x^2 - 9\right)$

F. If $y = \sin^{-1}\sqrt{x}$, find $\frac{dy}{dx}$

G. If $y = \log(\cosh 2x)$, find $\frac{dy}{dx}$

9.

◆A. Find dy and Δy of $y = f(x) = x^2 + x$ at $x = 10$ when $\Delta x = 0.1$

◆B. If the radius of a sphere is increased from 7 cm to 7.02 cm then find the approximate increase in the volume of the sphere.

◆C. If the increase in the side of a square is 2% then find the approximate percentage of increase in its area.

◆D. Show that the length of the subnormal at any point on the curve $y^2 = 4ax$ is constant

◆E. Show that the length of the subtangent at any point on the curve $y = a^x\ (a > 0)$ is a constant

◆F. Find dy and Δy if $y = \frac{1}{x+2}$, $x = 8$ and $\Delta x = 0.02$

◆G. Find the approximate value of $\sqrt[3]{999}$

H. The side of a square is increased from 3 cm to 3.01cm. Find the approximate increase in the area of the square .

I. The diameter of a sphere is measured to be 40 cm. If the error of 0.02 cm is made in it, then find the approximate errors of 0.02cm is made in it, then find the approximate errors in volume and surface area of the sphere.

J. Find the slope of the tangent to the curve $y=\dfrac{x-1}{x-2}(x\neq 2)$ at $x=10$

10.

◆A. Find the value of k, so that the length of the subnormal at any point on the curve $y=a^{1-k}.x^{k}$ is a constant

◆B. Show that the length of the subnormal at any point on the curve $xy=a^{2}$ varies as the cube of the ordinate of the point.

◆C. Let $f(x)=(x-1)(x-2)(x-3)$. Prove that there is more than one $'c'$ in $(1,3)$ such that $f'(c)=0$

◆D. Verify Rolle's theorem for the function $y=f(x)=x^{2}+4$ in $[-3,3]$

◆E. Verify Rolle's theorem for the function $f(x)=x(x+3)e^{-x/2}$ in $[-3,0]$

◆F. Find the intervals on which the function $f(x)=x^{3}+5x^{2}-8x+1$ is a strictly increasing function

G. Find the rate of change of area of a circle w.r.t radius when $r=5cm$

H. The distance – time formula for the motion of a particle along a straight line is $S=t^{3}-9t^{2}+24t-18$. Find when and where the velocity is zero.

I. Find the intervals on which $f(x)=x^{2}-3x+8$ is increasing or decreasing

J. Find the average rate of the change of $S=f(t)=2t^{2}+3$ between $t=2$ and $t=4$

SHORT ANSWER QUESTIONS

11.

◆A. Find the equation of the locus of P, if the ratio of the distances from P to A (5, -4) and B (7, 6) is 2 : 3.

◆B. Find the equation of locus of a point P such that the distance of P from origin is twice the distance of P from A (1, 2)

◆C. Find the equation of locus of P, if the line segment joining (2, 3) and (-1, 5) subtends a right angle at P.

◆D. Find the equation of locus of a point, the difference of whose distances from (-5, 0) and (5, 0) is 8.

◆E. Find the equation of the locus of a point, the sum of whose distances from (0, 2) and (0, –2) is 6.

◆F. The ends of the hypotenuse of a right angled triangle are (0, 6) and (6, 0). Find the equation of the locus of its third vertex.

◆G. A (5, 3) and B (3, –2) are two fixed points. Find the equation of the locus of P, so that the area of triangle PAB is 9.

◆H. A (1, 2), B (2, -3) and C (-2, 3) are three points. A point P moves such that $PA^2 + PB^2 = 2PC^2$. Show that the equation to the locus of P is $7x - 7y + 4 = 0$.

I. Find the equation of the locus of P, if A = (4, 0), B = (–4, 0) and $|PA - PB| = 4$.

J. A (2, 3), B (-3, 4) be two given points. Find the equation of locus of P so that the area of the triangle PAB is 8.5

K. Find the locus of the third vertex of a right angled triangle, the ends of whose hypotenuse are (4, 0) and (0, 4).

L. Find the equation of locus of P, if A (2, 3), B (2, -3) and $PA + PB = 8$.

12.

◆A. Show that the axes are to be rotated through an angle of $\frac{1}{2}Tan^{-1}\left(\frac{2h}{a-b}\right)$ so as to remove the xy term from the equation $ax^2 + 2hxy + by^2 = 0$ if $a \neq b$, and through an angle $\frac{\pi}{4}$ if $a = b$.

◆B. When the origin is shifted to the point (2, 3), the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$. Find the original equation of the curve.

◆C. When the axes are rotated through an angle 45^0, the transformed equation of a curve is $17x^2 - 16xy + 17y^2 = 225$. Find the original equation of the curve.

◆D. When the axes are rotated through an angle $\frac{\pi}{6}$, find the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$.

◆E. When the axes are rotated through an angle $\frac{\pi}{4}$, find the transformed equation of $3x^2 + 10xy + 3y^2 = 9$.

◆F. When the axes are rotated through an angle α, find the transformed equation of $x\cos\alpha + y\sin\alpha = p$.

G. Find the point to which the origin is to be shifted by the translation of axes so as to remove the first degree terms from the eq. $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, where $h^2 \neq ab$.

H. When the origin is shifted to (–1, 2) by the translation of axes, find the transformed equations of the following.

(i) $x^2 + y^2 + 2x - 4y + 1 = 0$ (ii) $2x^2 + y^2 - 4x + 4y = 0$

13.

◆A. Transform the equation $3x+4y+12=0$ into

(i) slope-intercept form (ii) intercept form (iii) normal form.

◆B. Transform the equation $\frac{x}{a}+\frac{y}{b}=1$ into the normal form when $a>0, b>0$. If the perpendicular distance of the straight line from the origin is p, deduce that $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

◆C. Find the value of p, if the following lines are concurrent

(i) $3x+4y=5,\ 2x+3y=4,\ px+4y=6$ (ii) $4x-3y-7=0,\ 2x+py+2=0,\ 6x+5y-1=0$

◆D. Find the value of k, if angle between the straight lines $4x-y+7=0$ and $kx-5y-9=0$ is $45°$.

◆E. A straight line Q (2, 3) makes an angle $\frac{3\pi}{4}$ with the negative direction of the X-axis. If the straight line intersects the line $x+y-7=0$ at P, find the distance PQ.

◆F. A straight line with slope 1 passes through $Q(-3,5)$ meets the line $x+y-6=0$ at P. Find the distance PQ.

◆G. Find the equation of straight line parallel to line $3x+4y=7$ and passing through the point of intersection of lines $x-2y-3=0$ and $x+3y-6=0$.

◆H. If P and Q are the lengths of the perpendiculars from the origin to the straight lines $x\sec\alpha+y\operatorname{cosec}\alpha=a$ and $x\cos\alpha-y\sin\alpha=a\cos2\alpha$. Prove that $4P^2+Q^2=a^2$.

◆I. Find the equations of the straight lines passing through the point (-3, 2) and making an angle of $45°$ with the straight line $3x-y+4=0$.

◆J. A variable straight line drawn through the point of intersection of the straight lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{b}+\frac{y}{a}=1$ meets the coordinate axes at A and B. Show that the locus of the midpoint of $\overline{AB}$ is $2(a+b)xy=ab(x+y)$.

K. A straight line L with negative slope passes through the point (8, 2) and cuts positive coordinate axes at the points P & Q. Find the minimum value of $OP + OQ$ as L varies, where O is the origin.

L. Each side of a square is of length 4 units. The centre of the square is (3, 7) and one of its diagonals is parallel to $y=x$. Find the coordinates of its vertices.

M. Find the area of the rhombus enclosed by the four straight lines $ax\pm by\pm c=0$.

N. If the straight lines $ax+by+c=0, bx+cy+a=0$ and $cx+ay+b=0$ are concurrent, then prove that $a^3+b^3+c^3=3abc$.

O. Find the point on the straight line $3x+y+4=0$, which is equidistant from (-5, 6) and (3, 2).

P. Find the points on the line $3x-4y-1=0$ which are at a distance of 5 units from the point $(3,2)$.

14.

◆A. Compute $\underset{x\to a}{Lt}\dfrac{x\sin a-a\sin x}{x-a}$

◆B. Compute $\underset{x\to 0}{Lt}\dfrac{\cos ax-\cos bx}{x^2}$

◆C. Check the continuity of the following function at '2'

$$f(x)=\begin{cases}\frac{1}{2}\left(x^2-4\right) & if\ \ 0<x<2\\ 0 & if\ \ x=2\\ 2-8x^{-3} & if\ \ x>2\end{cases}$$

◆D. Check the continuity of f given by

$$f(x)=\begin{cases}\dfrac{x^2-9}{x^2-2x-3} & if\ 0<x<5\ \ and\ x\neq 3\\ 1.5 & if\ \ x=3\end{cases}$$ at the point 3

◆E. Show that $f(x)=\begin{cases}\dfrac{\cos ax-\cos bx}{x^2} & if\ x\neq 0\\ \dfrac{1}{2}\left(b^2-a^2\right) & if\ x=0\end{cases}$ where a and b are real constants, is continuous at 0.

◆F. Compute $\underset{x\to 0}{Lt}\dfrac{(1+x)^{1/8}-(1-x)^{1/8}}{x}$

◆G. Find real constants a,b so that the function f given by

$$f(x)=\begin{cases}\sin x & if\ \ x\leq 0\\ x^2+a & if\ \ 0<x<1\\ bx+3 & if\ \ 1\leq x\leq 3\\ -3 & if\ \ x>3\end{cases}$$ is continuous on $\mathbb{R}$

H. Compute $\underset{x\to 1}{Lt}\dfrac{(2x-1)\left(\sqrt{x}-1\right)}{2x^2+x-3}$

I. Check the continuity of the function f given below at 1 and 2

$$f(x)=\begin{cases}x+1 & if\ \ x\leq 1\\ 2x & if\ \ 1<x<2\\ 1+x^2 & if\ \ x\geq 2\end{cases}$$

J. If f, given by $f(x)=\begin{cases}kx^2-k & ,\ if\ x\geq 1\\ 2 & ,\ if\ x<1\end{cases}$ is a continuous function on $\mathbb{R}$, then find the value of k.

K. Check the continuity of f given by $f(x)=\begin{cases}4-x^2 & if\ \ x\leq 0\\ x-5 & if\ \ 0<x\leq 1\\ 4x^2-9 & if\ \ 1<x<2\\ 3x+4 & if\ \ x\geq 2\end{cases}$

15.

◆A. Find the derivative of the following functions from first principles (4 marks each)

(i) $\sqrt{x+1}$ (ii) ax^2+bx+c (iii) $\sin 2x$ (iv) $\cos ax$

(v) $x\sin x$ (vi) $\tan 2x$ (vii) $\sec 3x$ (viii) $\cos^2 x$

◆B. If $x^y = e^{x-y}$ then show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

◆C. Find $\frac{dy}{dx}$ if $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$

◆D. If $x^{2/3}+y^{2/3}=a^{2/3}$ then prove that $\frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$

◆E. If $y=\tan^{-1}\sqrt{\frac{1-x}{1+x}}$ $(|x|<1)$ find $\frac{dy}{dx}$

◆F. If $y=\tan^{-1}\left(\frac{2x}{1-x^2}\right)(|x|<1)$ find $\frac{dy}{dx}$

◆G. If $\sin y = x\sin(a+y)$, then show that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ $(a \neq n\pi)$

◆H. Find the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

◆I. Find $\frac{dy}{dx}$ if $x = a\left(\frac{1-t^2}{1+t^2}\right)$, $y=\frac{2bt}{1+t^2}$

◆J. Differentiate $f(x)$ with respect to $g(x)$ for $f(x)=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$, $g(x)=\sqrt{1-x^2}$

K. Show that the function $f(x)=|x|+|x-1|, x\in\mathbb{R}$ is differentiable for all real numbers except for 0 & 1

L. Show that $y = x+\tan x$ satisfies $\cos^2 x\frac{d^2y}{dx^2}+2x=2y$

M. If $y=ax^{n+1}+bx^{-n}$ then prove that $x^2y''=n(n+1)y$

N. If $ay^4=(x+b)^5$ then prove that $5yy''=(y')^2$

16.

◆A. If the slope of the tangent to the curve $x^2-2xy+4y=0$ at a point on it is $-3/2$, then find the equations of the tangent and normal at that point.

◆B. Show that the tangent at any point θ on the curve $x=c\sec\theta$, $y=c\tan\theta$ is given by $y\sin\theta = x - c\cos\theta$.

◆C. Find the equations of tangent and normal to the curve $xy=10$ at $(2,5)$.

◆D. Show that at any point (x,y) on the curve $y=b\,e^{\frac{x}{a}}$, the length of the subtangent is constant and the length of the subnormal is $\frac{y^2}{a}$.

E. Show that the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2\ (a\neq 0, b\neq 0)$ at the point (a,b) is $\frac{x}{a}+\frac{y}{b}=2$

F. Find the value of k so that the length of the subnormal at any point on the curve $xy^k=a^{k+1}$ is a constant

G. Find the angle between the curve $2y=e^{-x/2}$ and $y-axis$.

H. Find the equations of the tangent and the normal to the curve $y=5x^4$ at the point $(1,5)$.

I. Find the lengths of subtangent and subnormal at a point on the curve $y=b\sin\frac{x}{a}$.

J. Find the equations of the tangent and normal to the curve $y^4=ax^3$ at (a,a)

K. Show that the curves $6x^2-5x+2y=0$ and $4x^2+8y^2=3$ touch each other at $\left(\frac{1}{2},\frac{1}{2}\right)$.

17.

◆A. The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of the edge is 10 centimeters .

◆B. A point P is moving on the curve $y=2x^2$. The x coordinate of P is increasing at the rate of 4 units per second. Find the rate at which the y coordinate is increasing when the point is at $(2,8)$.

◆C. A particle is moving in a straight line so that after t seconds its distance is s (in cms) from a fixed point on the line is given by $s=f(t)=8t+t^3$. Find (i) the velocity at time $t=2$ sec (ii) the initial velocity (iii) acceleration at $t=2$ sec.

◆D. The volume of a cube is increasing at the rate of 8cm^3/sec. How fast is the surface area increasing when the length of an edge is 12 cm?

◆E. A container in the shape of an inverted cone has height 12 cm and radius 6 cm at the top. If it is filled with water at the rate of 12cm^3/sec, what is the rate of change in the height of water level when the tank is filled 8 cm

F. A stone is dropped intro a quiet lake and ripples move in circles at the speed of 5cm/sec. At the instant when the radius of circular ripple is 8cm, how fast is the enclosed arc increases.

G. A container is in the shape of an inverted cone has height 8m and radius 6m at the top. If it is filled with water at the rate of 2m^3/minute, how fast is the height of water changing when the level is 4m?

H. The total cost $C(x)$ in rupees associated with production of x units of an item is given by $C(x)=0.005\,x^3-0.02x^2+30x+500$. Find the marginal cost when 3 units are produced (marginal cost is the rate of change of total cost).

I. A particle is moving along a line according to $s=f(t)=4t^3-3t^2+5t-1$ where s is measured in meters and t is measured in seconds. Find the velocity and acceleration at time t. At what time the acceleration is zero.

LONG ANSWER QUESTIONS

18.

◆A. If $Q(h,k)$ is the foot of the perpendicular from $P(x_1,y_1)$ on the straight line $ax+by+c=0$, then show that $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$.

And hence find the foot of the perpendicular from (–1, 3) on the straight line $5x-y-18=0$.

◆B. If $Q(h,k)$ is the image of the point $P(x_1,y_1)$ w.r.t the straight line $ax+by+c=0$ then show that $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$. And hence find the image of (1, –2) w.r.t the straight line $2x-3y+5=0$.

◆C. Find the circumcentre of the triangle whose vertices are (1, 3), (–3, 5) and (5, –1).

◆D. Find the circumcentre of the triangle whose sides are $3x-y-5=0, x+2y-4=0$ and $5x+3y+1=0$.

◆E. Find the orthocenter of the triangle whose vertices are (–5, –7), (13, 2) and (–5, 6).

◆F. If the equations of the sides of a triangle are $7x+y-10=0, x-2y+5=0$ and $x+y+2=0$, find the orthocenter of the triangle.

◆G. Two adjacent sides of a parallelogram are given by $4x+5y=0$ and $7x+2y=0$ and one diagonal is $11x+7y=9$. Find the equations of the remaining sides and the other diagonal.

◆H. The base of an equilateral triangle is $x+y-2=0$ and the opposite vertex is $(2,-1)$. Find the equations of the remaining sides.

◆I. If p and q are the lengths of the perpendiculars from the origin to the straight lines $x\sec\alpha+y\operatorname{cosec}\alpha=a$ and $x\cos\alpha-y\sin\alpha=a\cos2\alpha$, prove that $4p^2+q^2=a^2$.

◆J. Find the equations of the straight lines passing through $(1,1)$ and which are at a distance of 3 units from $(-2,3)$

K. Find the circumcentre of the triangle formed by the straight lines $x+y=0$, $2x+y+5=0$ and $x-y=2$.

L. Find the orthocenter of the triangle with the vertices (-2, -1) (6, -1) and (2, 5).

M. Find the orthocenter of the triangle formed by the lines $x+2y=0$, $4x+3y-5=0$ and $3x+y=0$.

N. Find the incentre of the triangle whose sides are $x+y-7=0, x-y+1=0$ and $x-3y+5=0$.

O. Find the circumcenter of the triangle whose vertices are (1, 3), (0, -2) and (-3, 1).

P. If the four straight lines $ax+by+p=0, ax+by+q=0, cx+dy+r=0$ and $cx+dy+s=0$ form a parallelogram, show that the area of the parallelogram so formed is $\left|\frac{(p-q)(r-s)}{bc-ad}\right|$

19.

◆A. Show that the product of perpendicular distances from a point (α,β) to the pair of straight lines $ax^2+2hxy+by^2=0$ is $\frac{\left|a\alpha^2+2h\alpha\beta+b\beta^2\right|}{\sqrt{(a-b)^2+4h^2}}$

◆B. If the equation $S \equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel straight lines then prove that (i) $h^2=ab$ (ii) $af^2=bg^2$ and

(iii) the distance between the parallel lines $=2\sqrt{\frac{g^2-ac}{a(a+b)}}=2\sqrt{\frac{f^2-bc}{b(a+b)}}$

◆C. If the second degree equation $S \equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ in the two variables x and y represents a pair of straight lines, then

(i) $abc+2fgh-af^2-bg^2-ch^2=0$ and (ii) $h^2 \ge ab, g^2 \ge ac$ and $f^2 \ge bc$.

◆D. If the equation $ax^2+2hxy+by^2=0$ represents a pair of intersecting lines, then the combined equation of the pair of bisectors of the angles between these lines is $h\left(x^2-y^2\right)=(a-b)xy$

◆E. Show that the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$ is $\left|\frac{n^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\right|$

◆F. Show that the straight lines represented by $3x^2+48xy+23y^2=0$ and $3x-2y+13=0$ form an equilateral triangles of area $\frac{13}{\sqrt{3}}$ sq. units

◆G. If the pairs of lines represented by $ax^2+2hxy+by^2=0$ and $ax^2+2hxy+by^2+2gx+2fy+c=0$ form a rhombus, prove that $(a-b)fg+h\left(f^2-g^2\right)=0$

◆H. Show that the product of the perpendicular distances from the origin to the pair of straight lines represented by $ax^2+2hxy+by^2+2gx+2fy+c=0$ is $\frac{|c|}{\sqrt{(a-b)^2+4h^2}}$

◆I. Show that the lines represented by $(lx+my)^2-3(mx-ly)^2=0$ and $lx+my+n=0$ form an equilateral triangle with area $\frac{n^2}{\sqrt{3}\left(l^2+m^2\right)}$.

J. Two equal sides of an isosceles triangle are $7x-y+3=0$ and $x+y-3=0$ and its third side passes through the point $(1,0)$. Find the equation of the third side

K. If (α,β) is the centroid of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my=1$.
Prove that $\frac{\alpha}{bl-hm}=\frac{\beta}{am-hl}=\frac{2}{3\left(bl^2-2hlm+am^2\right)}$

L. If two of the sides of a parallelogram are represented by $ax^2+2hxy+by^2=0$ and $px+qy=1$ is one of its diagonals. Prove that other diagonal is $y(bp-hq)=x(aq-hp)$

M. If the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of intersecting lines then show that the square of the distance of their point of intersection from the origin is $\frac{c(a+b)-f^2-g^2}{ab-h^2}$. Also show that the square of this distance is $\frac{f^2+g^2}{h^2+b^2}$ if the given lines are perpendicular.

N. Let the equation $ax^2+2hxy+by^2=0$ represents a pair of straight lines. Then the angle θ between the lines is given by $\cos\theta=\frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}$.

20.

◆A. Show that the lines joining the origin to the points of intersection of the curve $x^2-xy+y^2+3x+3y-2=0$ and the straight line $x-y-\sqrt{2}=0$ are mutually perpendicular

◆B. Find the values of k, if the lines joining the origin to the point of intersection of the curve $2x^2-2xy+3y^2+2x-y-1=0$ and the line $x+2y=k$ are mutually perpendicular

◆C. Find the angle between the lines joining the origin to the points of intersection of the curve $x^2+2xy+y^2+2x+2y-5=0$ and the line $3x-y+1=0$

◆D. Find the condition for the chord $lx+my=1$ of the circle $x^2+y^2=a^2$ to subtend a right angle at the origin

◆E. Find the lines joining the origin to the points of intersection of the curve $7x^2-4xy+8y^2+2x-4y-8=0$ with the straight line $3x-y=2$ and also the angle between them.

F. Find the condition for the lines joining the origin to the points of intersection of the circle $x^2+y^2=a^2$ and the line $lx+my=1$ to coincide

G. Write down the equation of the pair of straight lines joining the origin to the points of intersection of the line $6x-y+8=0$ with the pair of straight lines $3x^2+4xy-4y^2-11x+2y+6=0$. Show that the lines so obtained make equal angles with coordinate axes

H. Show that the equation $2x^2-13xy-7y^2+x+23y-6=0$ represents a pair of straight lines. Also find the angles between them and the coordinates of the point of intersection of the lines.

21.

◆A. Show that the lines whose d.c's are given by $l+m+n=0$, $2mn+3nl-5lm=0$ are perpendicular to each other.

◆B. Find the angle between the lines whose direction cosines satisfy the equations $l+m+n=0$, $l^2+m^2-n^2=0$.

◆C. If a ray makes angles $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube find $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta$.

◆D. Find the direction cosines of two lines which are connected by the relations $l-5m+3n=0$ and $7l^2+5m^2-3n^2=0$.

◆E. Find the direction cosines of two lines which are connected by the relations $l+m+n=0$ and $mn-2nl-2lm=0$

◆F. Find the angle between the lines whose direction cosines are given by the equations $3l+m+5n=0$ and $6mn-2nl+5lm=0$.

G. If a variable line in two adjacent positions has direction cosines (l,m,n) and $(l+\delta l, m+\delta m, n+\delta n)$, show that the small angle $\delta\theta$ between the two positions is given by $(\delta\theta)^2=(\delta l)^2+(\delta m)^2+(\delta n)^2$.

H. Find the angle between two diagonals of a cube.

I. The vertices of triangle are $A(1,4,2), B(-2,1,2), C(2,3,-4)$. Find $\underline{|A}, \underline{|B}, \underline{|C}$.

22.

◆A. If $y=\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ for $0<|x|<1$, find $\frac{dy}{dx}$

◆B. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

◆C. If $y=x\sqrt{a^2+x^2}+a^2\log\left(x+\sqrt{a^2+x^2}\right)$ then prove that $\frac{dy}{dx}=2\sqrt{a^2+x^2}$

◆D. If $y=x^{\tan x}+(\sin x)^{\cos x}$, find $\frac{dy}{dx}$

◆E. Find the derivative of the function $(\sin x)^{\log x}+x^{\sin x}$.

◆F. If $x^y + y^x = a^b$ then prove that $\frac{dy}{dx} = -\left[\frac{y\,x^{y-1} + y^x \log y}{x^y \log x + x\,y^{x-1}}\right]$

◆G. If $f(x) = \sin^{-1}\sqrt{\frac{x-\beta}{\alpha-\beta}}$ and $g(x) = \tan^{-1}\sqrt{\frac{x-\beta}{\alpha-x}}$ then Prove $f'(x) = g'(x)$ $(\beta < x < \alpha)$

◆H. If $a > b > 0$ and $0 < x < \pi$, $f(x) = \left(a^2 - b^2\right)^{-1/2} \cos^{-1}\left(\frac{a\cos x + b}{a - b\cos x}\right)$ then $f'(x) = (a + b\cos x)^{-1}$

I. Find $\frac{dy}{dx}$ if $y = \frac{(1-2x)^{2/3}(1+3x)^{-3/4}}{(1-6x)^{5/6}(1+7x)^{-6/7}}$

J. If $x = a(t - \sin t)$, $y = a(1 + \cos t)$ find $\frac{d^2y}{dx^2}$

K. If $y = e^{-\frac{k}{2}x}(a\cos nx + b\sin nx)$ then prove that $y'' + ky' + \left(n^2 + \frac{k^2}{4}\right)y = 0$

L. Differentiate $f(x)$ with respect to $g(x)$ where $f(x) = Tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $g(x) = Tan^{-1}x$.

M. If $y = a\cos(\sin x) + b\sin(\sin x)$ then prove $y'' + (\tan x)y' + y\cos^2 x = 0$.

23.

◆A. If the tangent at any point on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ intersects the coordinates axes in A and B, then show that the length AB is constant.

◆B. If the tangent at any point P on the curve $x^m y^n = a^{m+n}$ $(mn \neq 0)$ meets the coordinate axes in A,B then show that $AP : BP$ is constant

◆C. Show that the curves $y^2 = 4(x+1)$ and $y^2 = 36(9-x)$ intersect orthogonally.

◆D. Find the angle between the curves $xy = 2$ and $x^2 + 4y = 0$

◆E. Find the angle between the curves $y^2 = 4x$ and $x^2 + y^2 = 5$

◆F. Show that the condition for orthogonality of the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ is $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$

◆G. Find the lengths of subtangent, subnormal at a point t on the curve $x = a(\cos t + t\sin t)$; $y = a(\sin t - t\cos t)$

H. At any point t on the curve $x = a(t + \sin t)$, $y = a(1 - \cos t)$. Find the lengths of tangent, normal, subangent and sub normal.

I. Find the equations of the tangents to the curve $y = 3x^2 - x^3$ where it meets the $x - axis$

J. Show that the tangent at $P(x_1, y_1)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $yy_1^{\frac{-1}{2}} + xx_1^{\frac{-1}{2}} = a^{\frac{1}{2}}$.

K. Show that the square of the length of subtangent at any point on the curve $by^2 = (x+a)^3 \ (b \neq 0)$ varies with the length of the subnormal at that point

L. Find the angle between the curves $y^2 = 8x$ and $4x^2 + y^2 = 32$

M. Find the angle between the curves $2y^2 - 9x = 0$ and $3x^2 + 4y = 0$ (in the 4th quadrant)

24.

◆A. Find two positive integers x and y such that $x + y = 60$ and xy^3 is maximum

◆B. From a rectangular sheet of dimensions $30\text{cm} \times 80\text{cm}$, four equal squares of side x cm are removed at the corners and the sides are then turned up so as to form an open rectangular box. Find the value of x, so that the volume of the box is the greatest.

◆C. A window is in the shape of a rectangle surmounted by a semi circle. If the perimeter of the window is 20 ft, find the maximum area.

◆D. If the curved surface of a right circular cylinder inscribed in a sphere of radius 'r' is maximum, show that the height of the cylinder is $\sqrt{2}r$

◆E. A wire of length L is cut into two parts which are bent respectively in the form of a square and a circle. What are the lengths of the pieces of the wire respectively so that the sum of the areas is the least.

F. Find two positive numbers whose sum is 15 so that the sum of their squares is minimum

G. Find the maximum area of the rectangle that can be formed with fixed perimeter 20

H. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

I. Find the absolute maximum and absolute minimum of $f(x) = 8x^3 + 81x^2 - 42x - 8$ on $[-8, 2]$

J. The profit function $p(x)$ of a company selling 'x' items per day is given by $P(x) = (150 - x)x - 1000$. Find the number of items that the company should manufacture to get maximum profit. Also find the maximum profit.

wish you all the best